$$n_b = \frac{\Omega_b \rho_c}{m_p}(1+z)^3 \tag{1}$$

$$\frac{dz}{dt} = -H_0\sqrt{y^2(1-\Omega_m+\Omega_m y^3)} \approx -\sqrt{\Omega_b}H_0 y^{5/2} \tag{2}$$

---

$$Error = \sqrt{\frac{2}{(2\ell+1)f_{cut}L}} \times (E_{CMB} + E_{detector}) \tag{3}$$

$$E_{detector} = f_{sky}\frac{4\pi s_X^2}{t_{obs}} \cdot e^{\ell^2 \cdot \sigma_b^2} \tag{4}$$

## 0.1. Intensity in terms of $\Omega_d$ and $u_*$

$$\begin{aligned}\Delta i_\nu \;=\; & \left.\frac{dB_\nu}{dT}\right|_{T_{CMB}} \times \\ & \left[\frac{3}{32\pi}\frac{c^4}{h}\frac{\rho_c}{H_0\sqrt{\Omega_m}}\frac{Q_{UV}}{a\rho_g}\left(\frac{h}{k_B}\right)^{4+\beta}\frac{\nu_r^\beta}{C_\beta}\frac{T_{CMB}^{-3-\beta}}{4+\beta}\right] \times \\ & \int_z^0 dz'\left(\frac{\nu}{\nu_r}\right)^{\beta_{\nu_e}}\left[\Omega_d(z')u_*(z')\right](1+z')^{1/2+\beta_{\nu_e}}\end{aligned} \tag{5}$$

where $T_{CMB}$ is the CMB temperature today, $\nu$ and $\nu_e$ are the observed and emitted frequency respectively. In the calculations we use $Q_{UV} = 1$.

If $\beta_{\nu_e} = \beta$, this simplifies to

$$\begin{aligned}\Delta i_\nu \;=\; & \nu^\beta \left.\frac{dB_\nu}{dT}\right|_{T_{CMB}} \times \\ & \left[\frac{3}{32\pi}\frac{c^4}{h}\frac{\rho_c}{H_0\sqrt{\Omega_m}}\frac{Q_{UV}}{a\rho_g}\left(\frac{h}{k_B}\right)^{4+\beta}\frac{1}{C_\beta}\frac{T_{CMB}^{-3-\beta}}{4+\beta}\right] \times \\ & \int_z^0 dz'\left[\Omega_d(z')u_*(z')\right](1+z')^{1/2+\beta}\end{aligned} \tag{6}$$

The total intensity per unit $z$ can then be written as

$$\frac{dI}{dz} = \frac{d}{dz}\int_0^\infty \Delta i_\nu d\nu = \frac{3\rho_c}{16\pi a\rho_g}\left[\Omega_d(z)u_*(z)\right]\frac{1}{1+z}\frac{dt}{dz} \tag{7}$$

which we note is independent of $\beta$ and therefore of $\lambda_r$.

## 0.2. PopIII Stars

Angular size of object with comoving size $D_c$ at a comoving distance $L_c$:

$$\frac{\pi}{\ell} \approx \theta[rad] = \frac{D_c}{L_c} = \frac{D_c}{c\cdot\int_{t_i}^{t_0}(1+z)dt} = \frac{D_c}{c\int_z^0 dz(1+z)\frac{dt}{dz}} \tag{8}$$

This means that the sphere of light emitted from a star at $z_i$ will expand as:

$$\theta_{bulb} = \frac{D_{bulb}}{L_c} = \frac{c\int_{z_i}^z dz(1+z)\frac{dt}{dz}}{c\int_z^0 dz(1+z)\frac{dt}{dz}} \tag{9}$$

The star density can be calculated as:

$$n_* = \frac{du_*}{dt}/L_* = \left[fE_\gamma\left(\frac{dn_\gamma}{dz}/n_b\right)n_b\right]\frac{dz}{dt}/L_* \tag{10}$$

where we quite arbitrary suppose that $L_* = 10^5 L_\odot$. This means that the average distance between two stars is given by

$$r_* \approx n_*^{-1/3} \tag{11}$$

which means that the angular distance between the stars will be

$$\theta_* = \frac{r_*}{c\int_z^0 dz(1+z)\frac{dt}{dz}} \tag{12}$$

## 0.3. Reionizing sphere

Reionization sphere radius from source at $z_i$:

$$\frac{dr_{ion}}{dt}(t) \;=\; \frac{L_*}{h\nu 4\pi n_b}\times r_{ion}(t)^{-2} \tag{13}$$

$$\frac{dr_{ion}}{dt}(z) \;=\; \frac{L_*}{h\nu 4\pi n_b}\times r_{ion}(z)^{-2} \tag{14}$$

if we suppose that all photons are ionizing and have an energy $h\nu$ =13.6 eV. Solving this (supposing that $r_{ion}(t_i) = 0$) we obtain the linear size of the sphere, $D_{ion} = 2r_{ion}$ (in comoving units):

$$D_{ion}(t) \;=\; 2\left[\frac{3L_*}{h\nu 4\pi(\Omega_b\rho_c/m_p)}t\right]^{1/3} \tag{15}$$

$$D_{ion}(z) \;=\; 2\left[\frac{3}{2}\cdot\frac{3L_*\cdot\left[(1+z_i)^{-3/2}-(1+z)^{-3/2}\right]}{h\nu 4\pi(\Omega_b\rho_c/m_p)}\right]^{1/3} \tag{16}$$

where we have supposed that $z \gg 2$ in the last step.

$$x \tag{17}$$