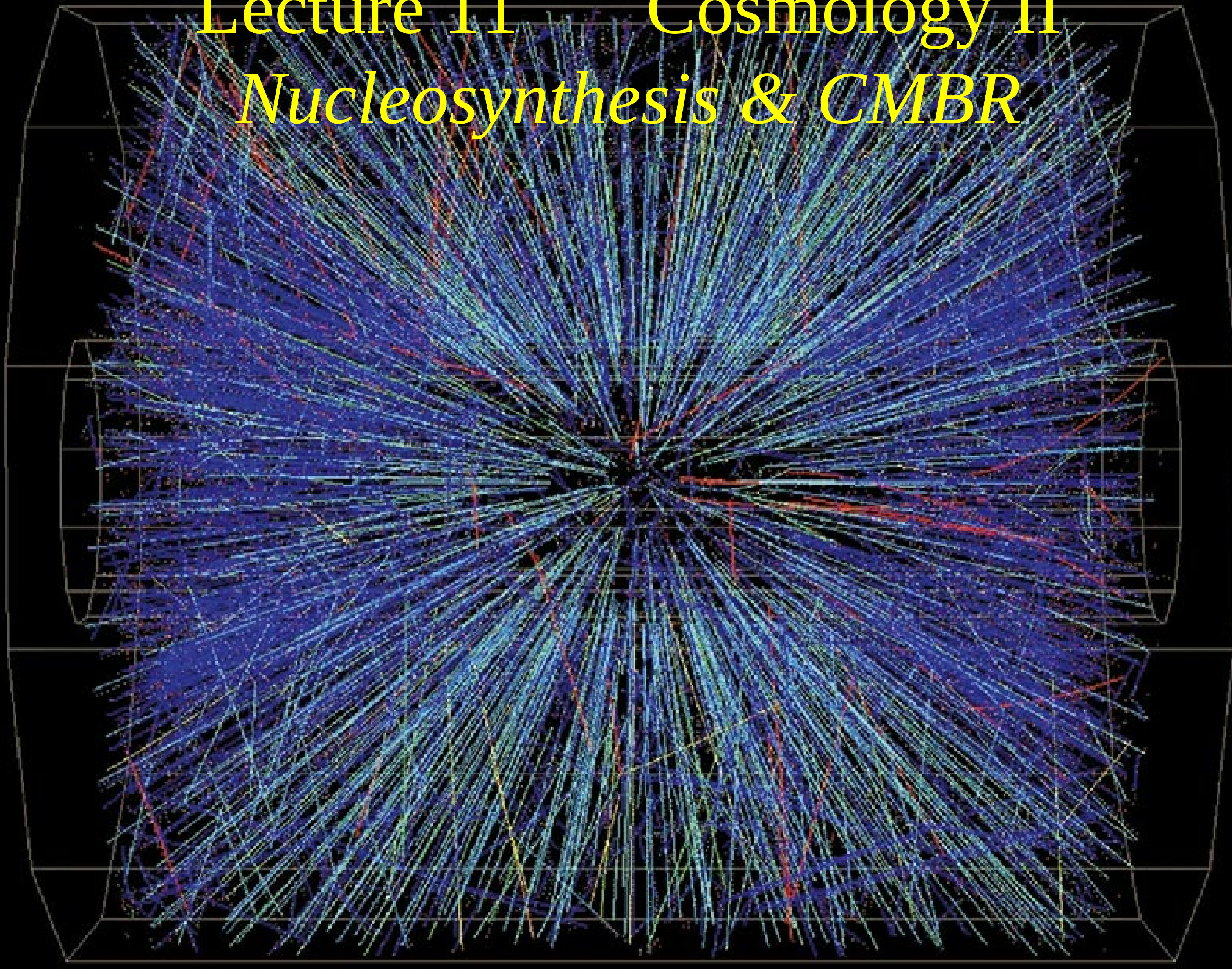
Lecture 11 Cosmology II

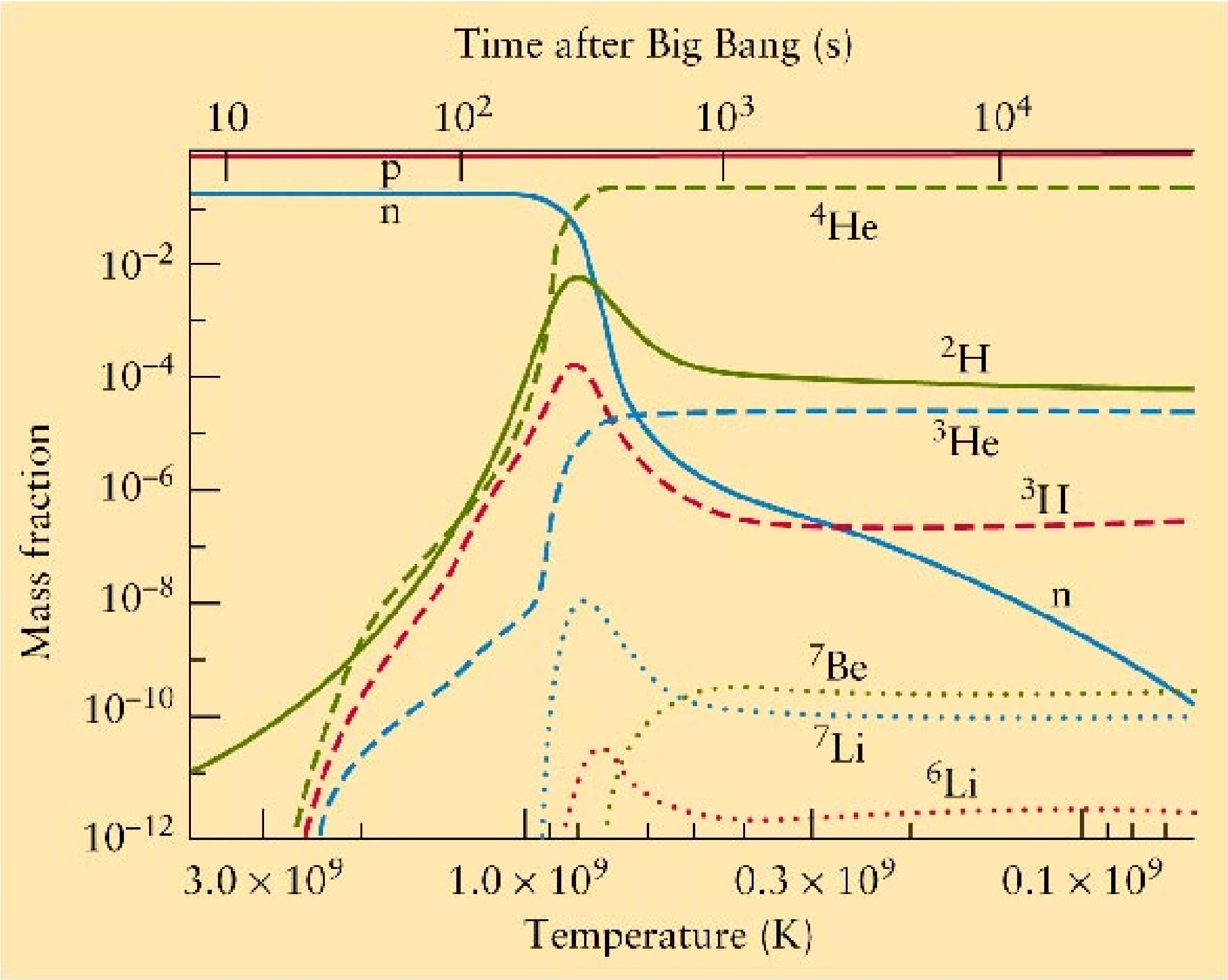
Nucleosynthesis & CMBR



Era of Nucleosynthesis (10^{-3} sec < t < 3 min)

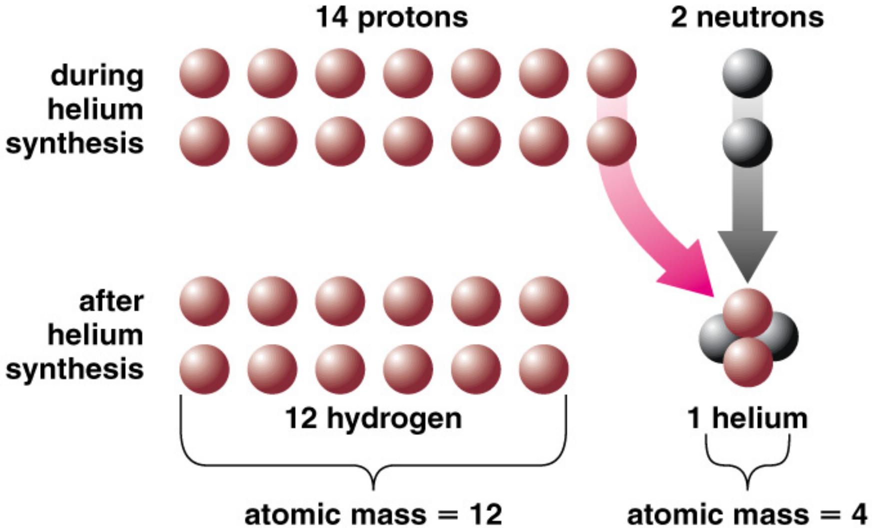
- During this era, protons & neutrons started fusing…
  - but new nuclei were also torn apart by the high temperatures
- When the Universe was 3 min old, it had cooled to 10^9 K.
  - at this point, the fusion stopped
- Afterwards, the baryonic matter leftover in the Universe was:
  - 75% Hydrogen nuclei (i.e. individual protons)
  - 25% Helium nuclei
  - trace amounts of Deuterium (H isotope) & Lithium nuclei

A background of neutrinos and most of the helium in the universe are relics of the primordial fireball.



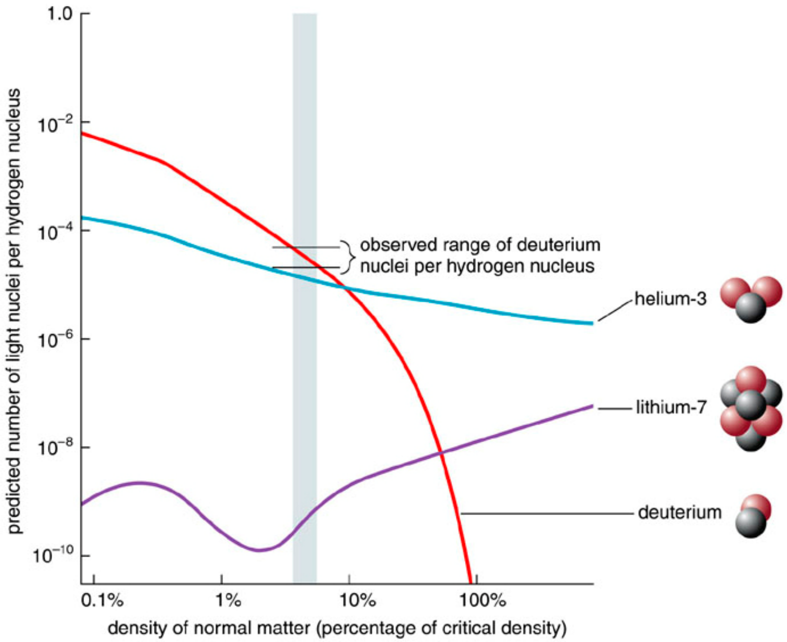
Cosmic Helium Abundance

- In the Era of Nucleosynthesis, i.e. the first three minutes
  - number of protons & neutrons roughly equal as long as T > 10^{11} K
  - below 10^{11} K, proton-to-neutron reactions no longer occur
    - neutrons still decay into protons
  - protons begin to outnumber neutrons
- At T < 10^{10} K, the products of fusion reactions no longer break up.
  - Helium, Deuterium, & Lithium remain stable
- At this time, Big Bang model predicts a 7-to-1 proton:neutron ratio.
- For every 2 n & 2 p^+ which fused into a Helium nucleus…
  - there are 12 p^+ or Hydrogen nuclei
- Model predicts a 3-to-1 H:He
- This what we observe:
  - minimum of 25% He in all galaxies



Abundances of Other Light Nuclei

- By the time stable ^{4}He formed…
  - the Universe was too cool for He to fuse into C or other heavier nuclei
  - ^{4}He could fuse with ^{3}H to form stable ^{7}Li
- Deuterium (^{2}H) is a “leftover” isotope.
  - if densities had been greater, fusion would have gone faster, and more neutrons would have ended up in ^{4}He instead of ^{2}H
  - nucleosynthesis models predict the amount of leftover ^{2}H for each density



- The measured abundance of ^{2}H is
  - one for every 40,000 H atoms
- Compared to the model calculations
  - the density of ordinary matter is 4% of the critical density.
- Density of matter appears to be more like 30% of the critical density.
- Majority of matter in the Universe is “exotic”, such as WIMPs.

Era of Nuclei (3 min < t < 3.8 x 10^5 yr)

- The Universe was a hot plasma of H & He nuclei and electrons.
  - photons bounced from electron to electron, not traveling very far
  - the Universe was opaque
- When the Universe was 380,000 yrs old…
  - it had cooled to a temperature of 3,000 K
  - electrons combined with nuclei to form stable atoms of H & He
  - the photons were free to stream across the Universe
  - the Universe became transparent

Era of Atoms ($3.8 \times 10^5 < t < 10^9$ yr)

- The Universe was filled with atomic gas.
  - sometimes referred to as the "Cosmic Dark Ages"
- Density enhancements in the gas and gravitational attraction by dark matter…
  - eventually form protogalactic clouds
  - the first star formation lights up the Universe
  - which provokes the formation of galaxies

Era of Galaxies ($t > 10^9$ yr)

- The first galaxies came into existence about 1 billion years after the Big Bang.
- This is the current era of the Universe.

Galaxies formed from density fluctuations in the early universe.

Jeans length for density fluctuations

$$L_J = [\pi kT / mG\rho_m]^{1/2}$$

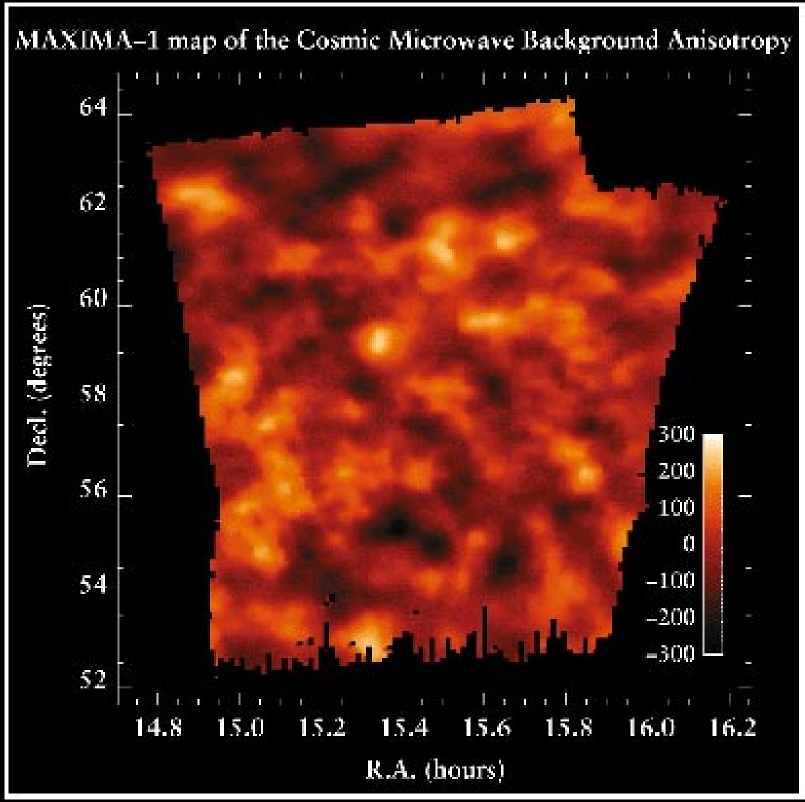
k = 1.38x10-23 J/K

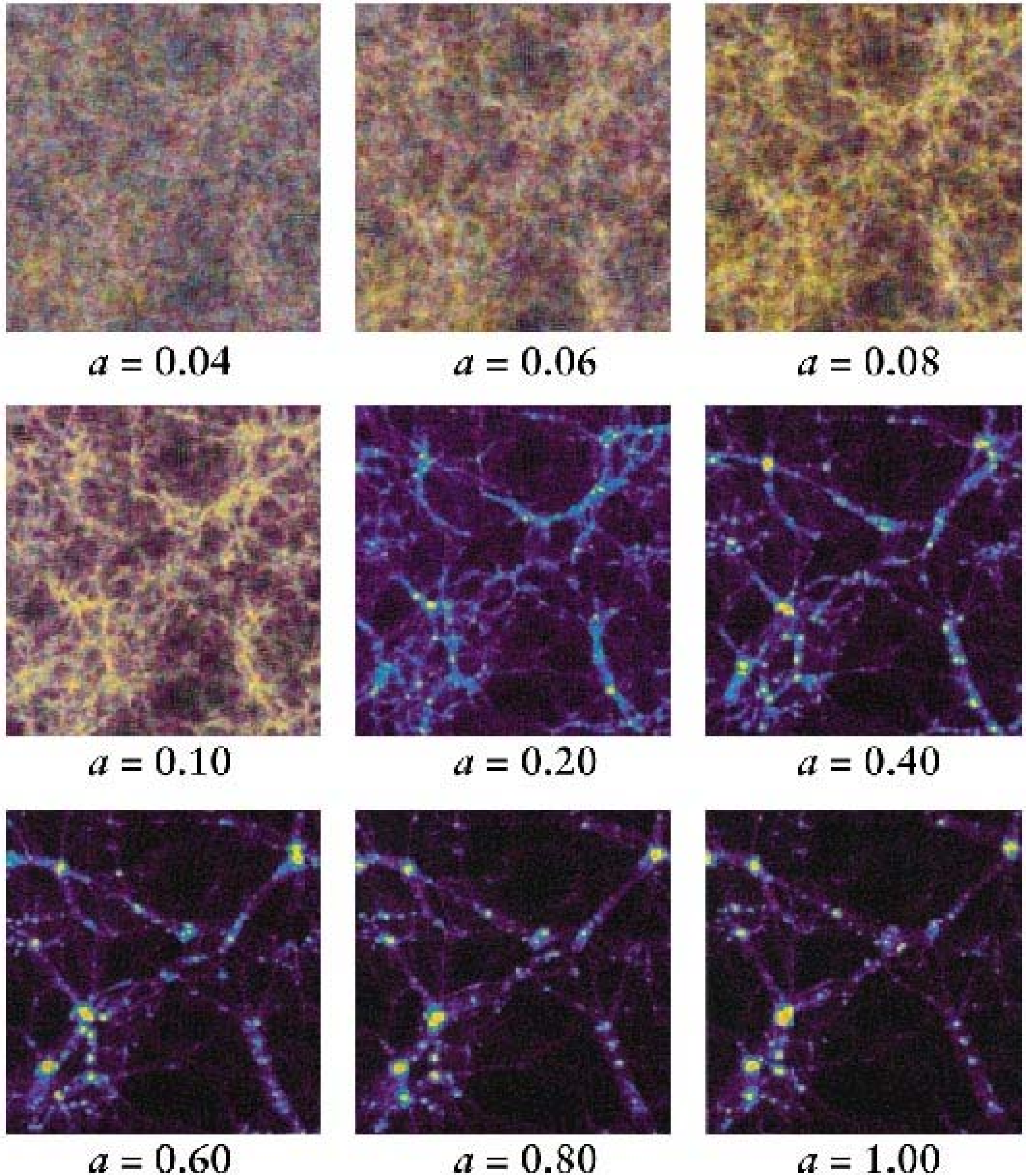
T = gas temperature

m = mass of a single particle in the gas

G = universal gravitation constant

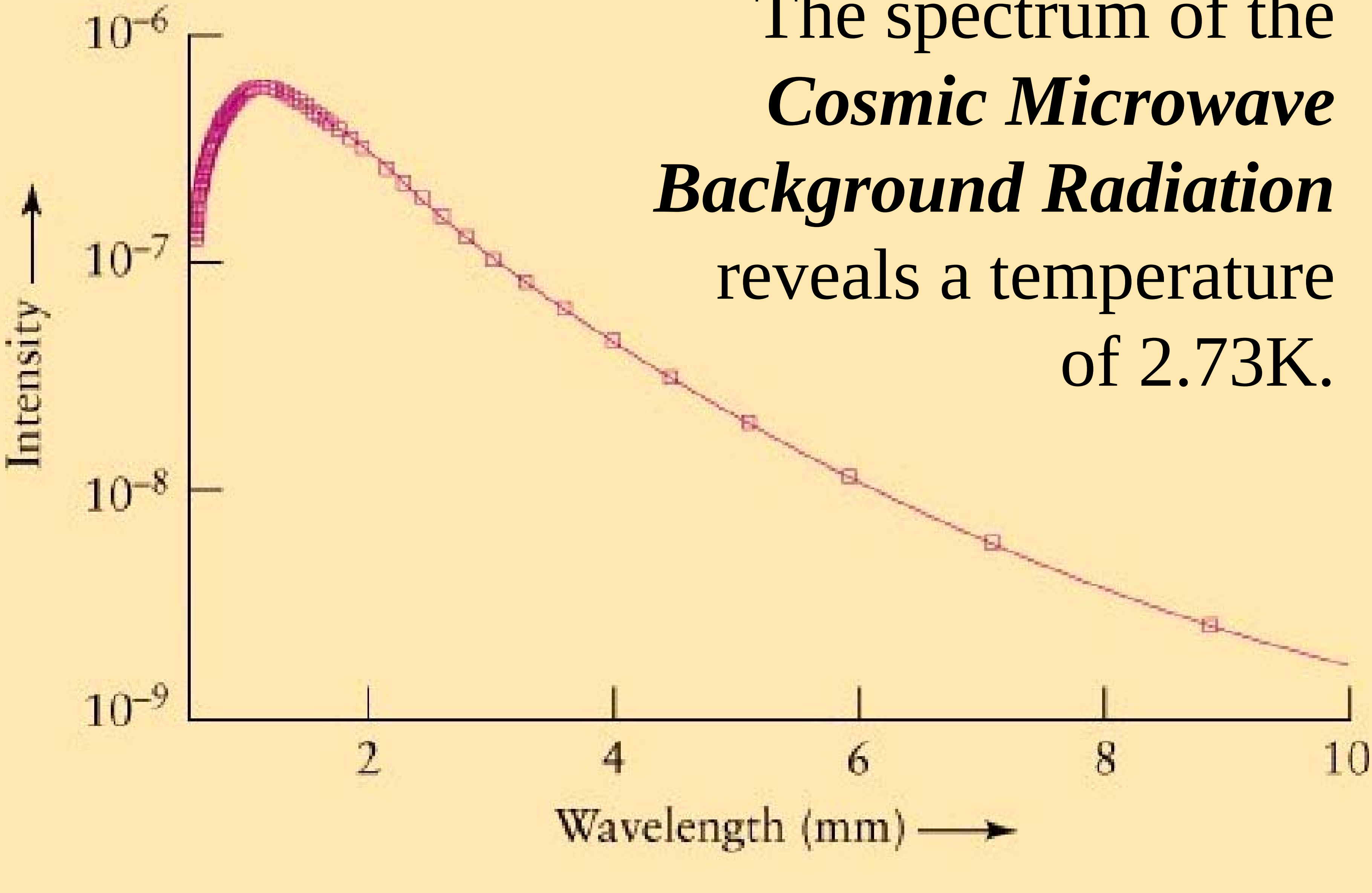
ρ_m = average density of matter in the gas



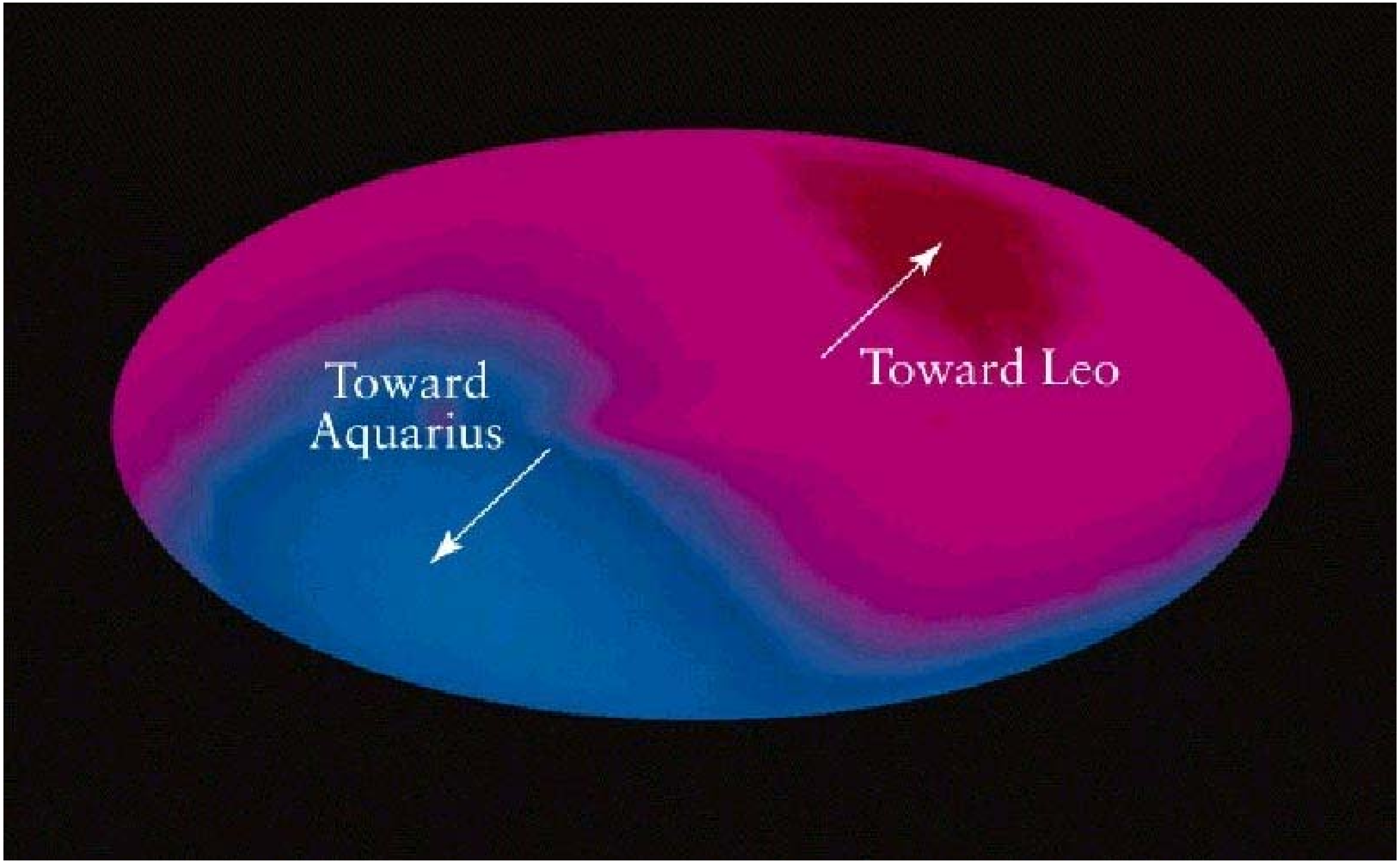

$a = 0.04$
$a = 0.06$
$a = 0.08$
$a = 0.10$
$a = 0.20$
$a = 0.40$
$a = 0.60$
$a = 0.80$
$a = 1.00$

The microwave radiation that fills all space is evidence of a hot Big Bang.

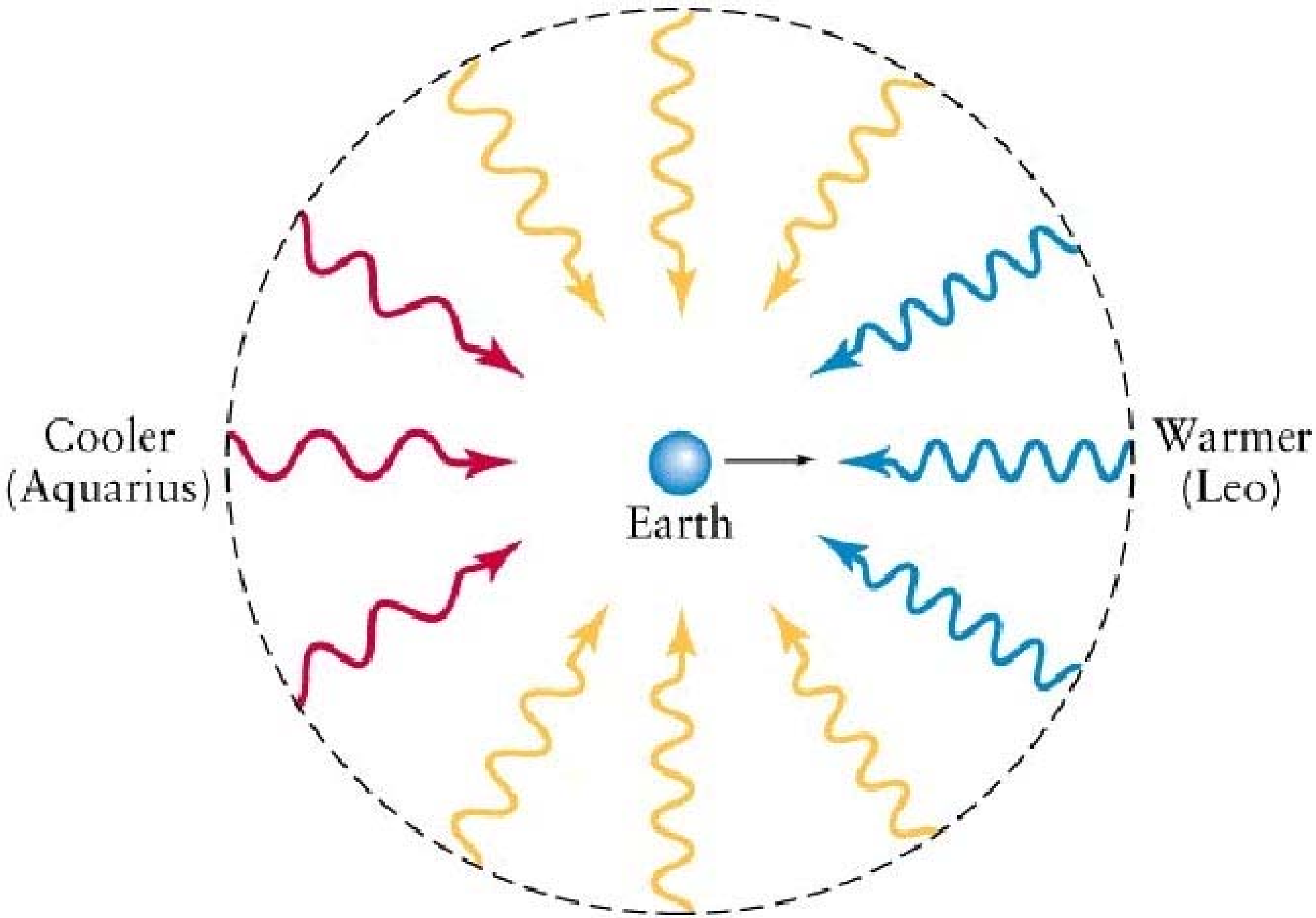
The spectrum of the ***Cosmic Microwave Background Radiation*** reveals a temperature of 2.73K.



Largest variations due to the motion of Earth through the cosmos



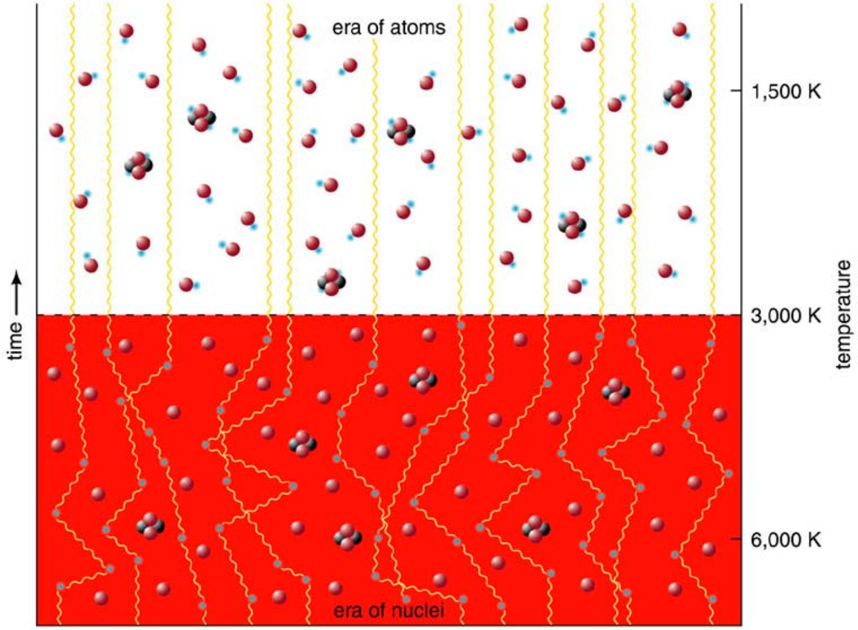
The microwave sky where variations are due to the motion of Earth through the cosmos.



Cosmic Microwave Background

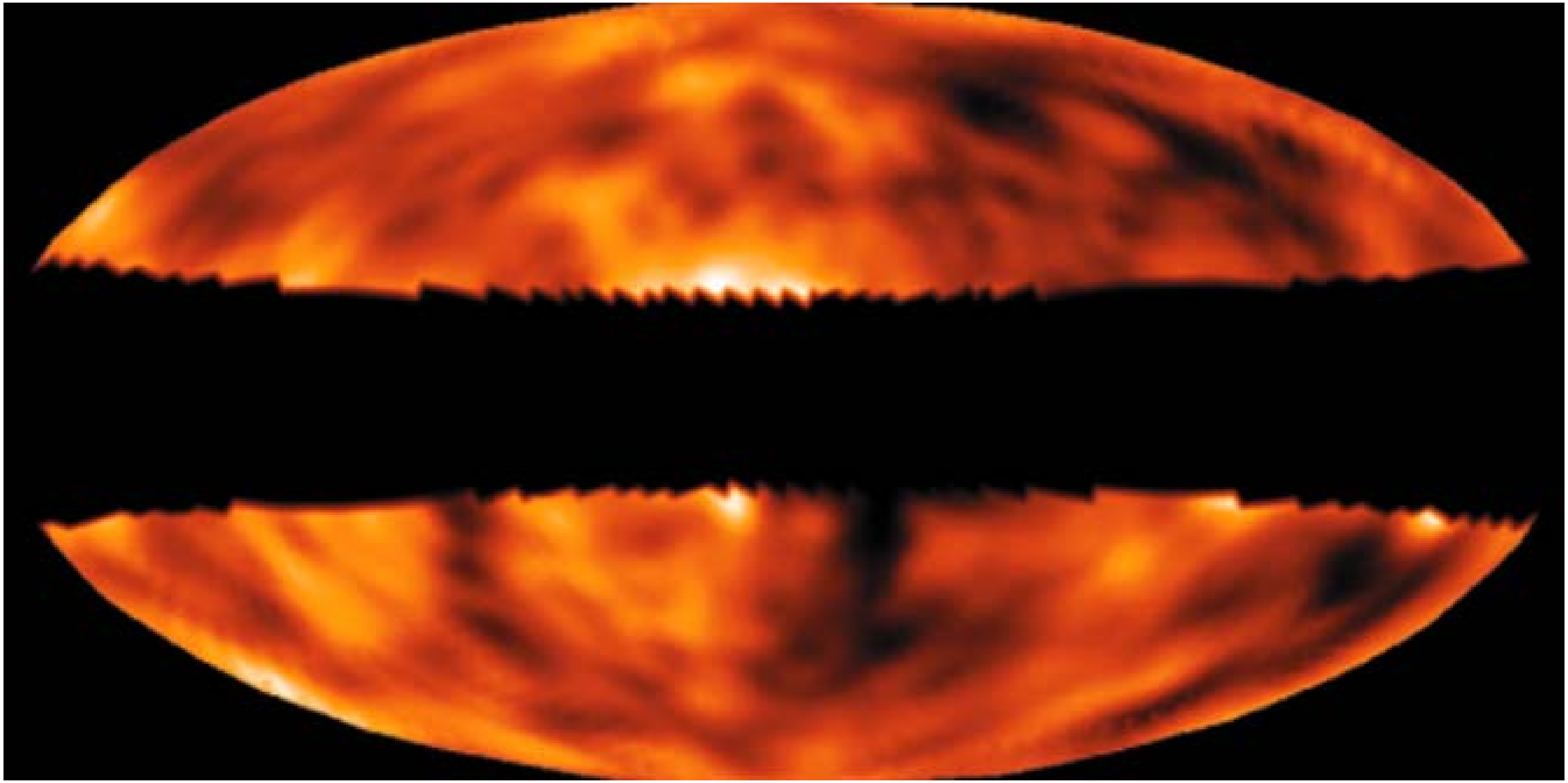
- The Universe is immersed in a sea of radiation.
- This is the same radiation which was unleashed at the end of the Era of Nuclei.
  - 380,000 years after the Big Bang, the Universe had cooled enough for free electrons to become bound into atoms of H & He
  - without electrons to scatter them, photons were able to travel unhindered throughout the Universe
  - the Universe became *transparent*

The temperature of the Universe was 3,000 K at this time.

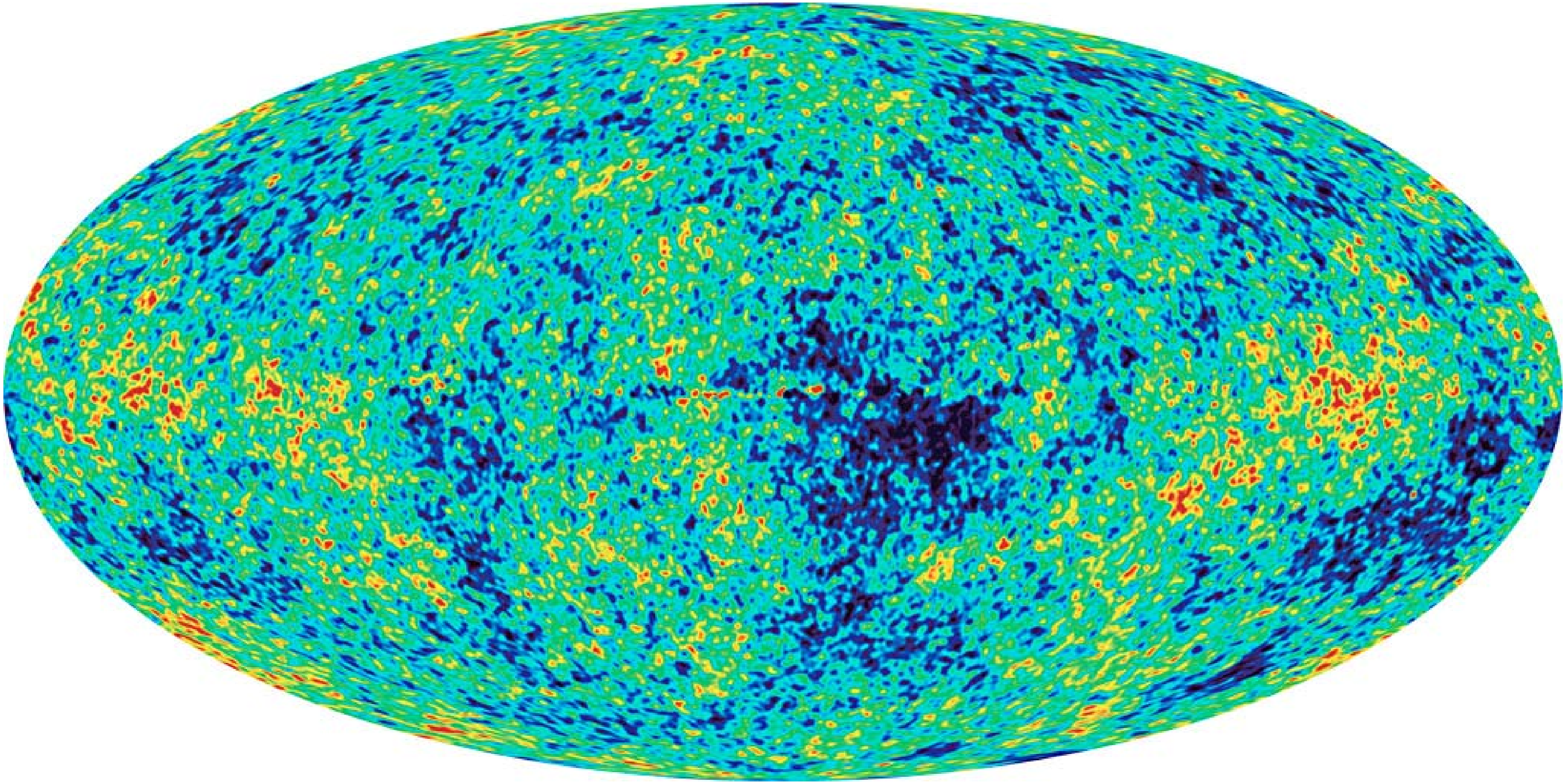


Cosmic Microwave Background…

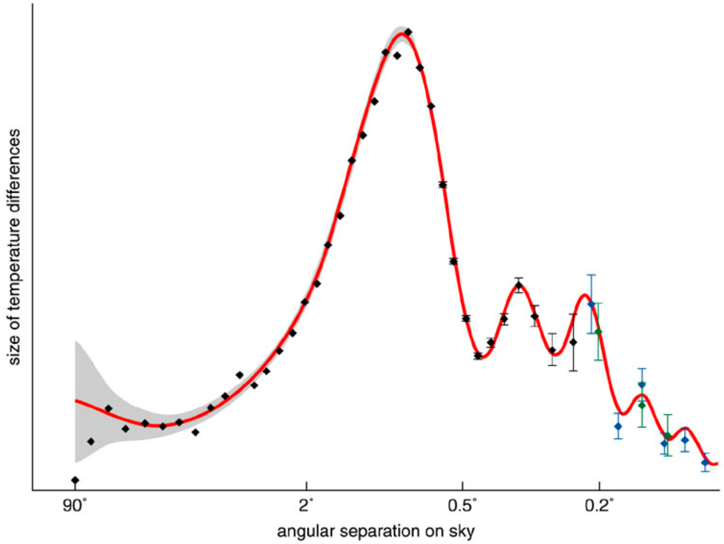
- …was mapped by the *COsmic Background Explorer* (COBE) in 1990s
- While very smooth and uniform across the sky…
- COBE did find slight temperature variations from place to place on the level of a few parts in 100,000.



- In 2003, the *Wilkinson Microwave Anisotropy Probe* (WMAP) measured the Cosmic Microwave Background with much more precision than COBE.
- It detected far more subtle, small-scale temperature variations.



- A Big Bang model with inflation was fitted to:
  - temperature variations plotted as angular separation on the sky
  - the data are shown here



- Overall geometry of the Universe is flat.
- Total matter density is 27% of the critical density.
  - in agreement with M/L in clusters of galaxies
- Density of baryonic (ordinary) matter is 4.4% of critical density.
  - in agreement with observed abundance of Deuterium
- Flat geometry + matter density < critical implies dark energy.
  - in agreement with accelerating expansion from white dwarf supernovae
- Age of the Universe is 13.7 billion years.

The metric of an isotropic and homogeneous Universe

- "Robertson-Walker metric"

$$ds^2 = (cdt)^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right)$$

- *R(t)* is the scale factor
- *k* is the curvature constant

  k=0: flat space

  k>0: spherical geometry

  k<0: hyperbolic geometry

Cosmological redshift

- While a photon travels from a distance source to an observer on Earth, the Universe expands in size from R_{then} to R_{now}.
- Not only the Universe itself expands, but also the wavelength of the photon λ.

$$\lambda_{received} = \frac{R_{now}}{R_{then}} \lambda_{emitted}$$

Cosmological redshift

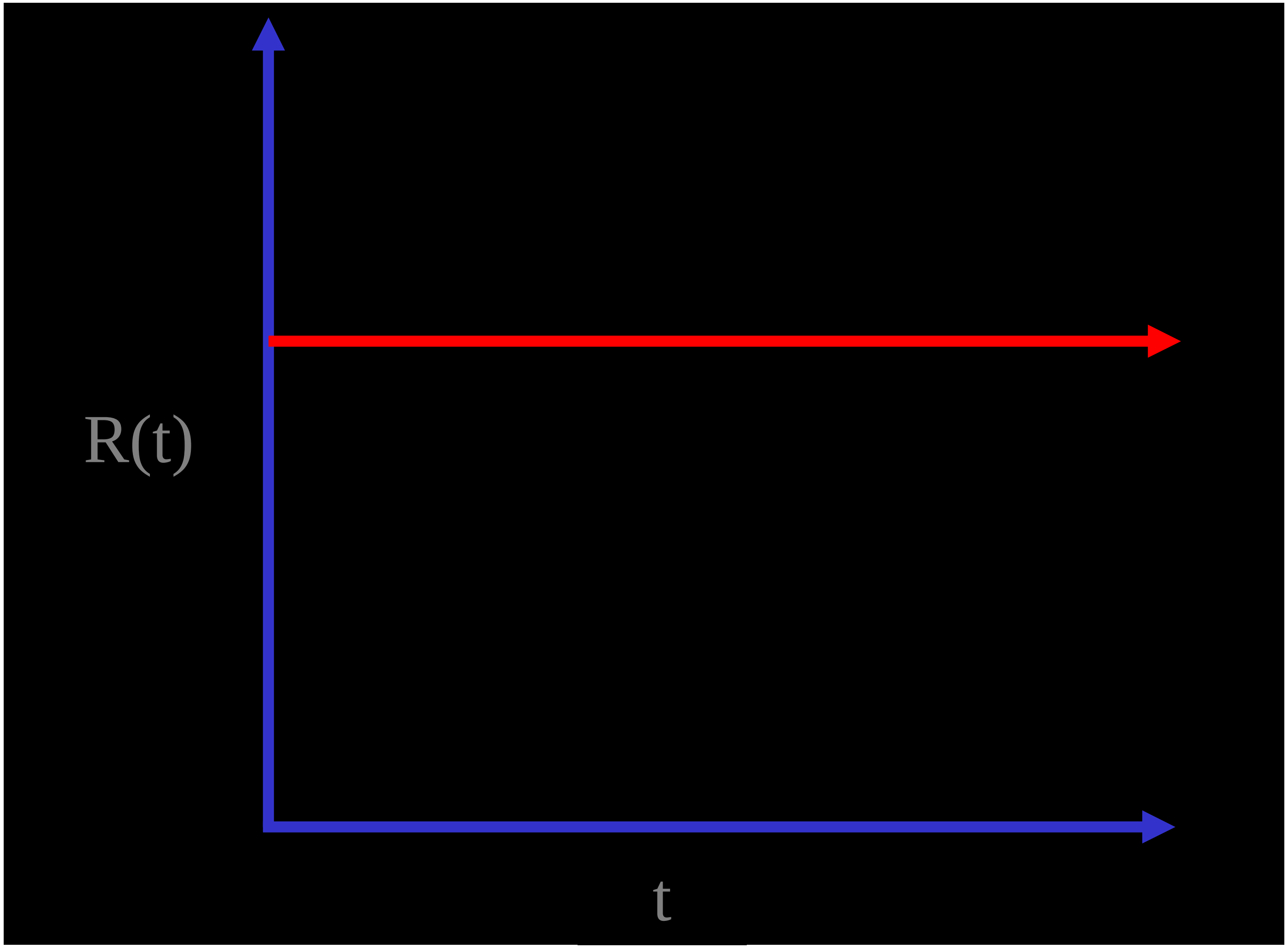
- General definition of redshift:

$$z = \frac{\lambda_{received} - \lambda_{emitted}}{\lambda_{emitted}}$$

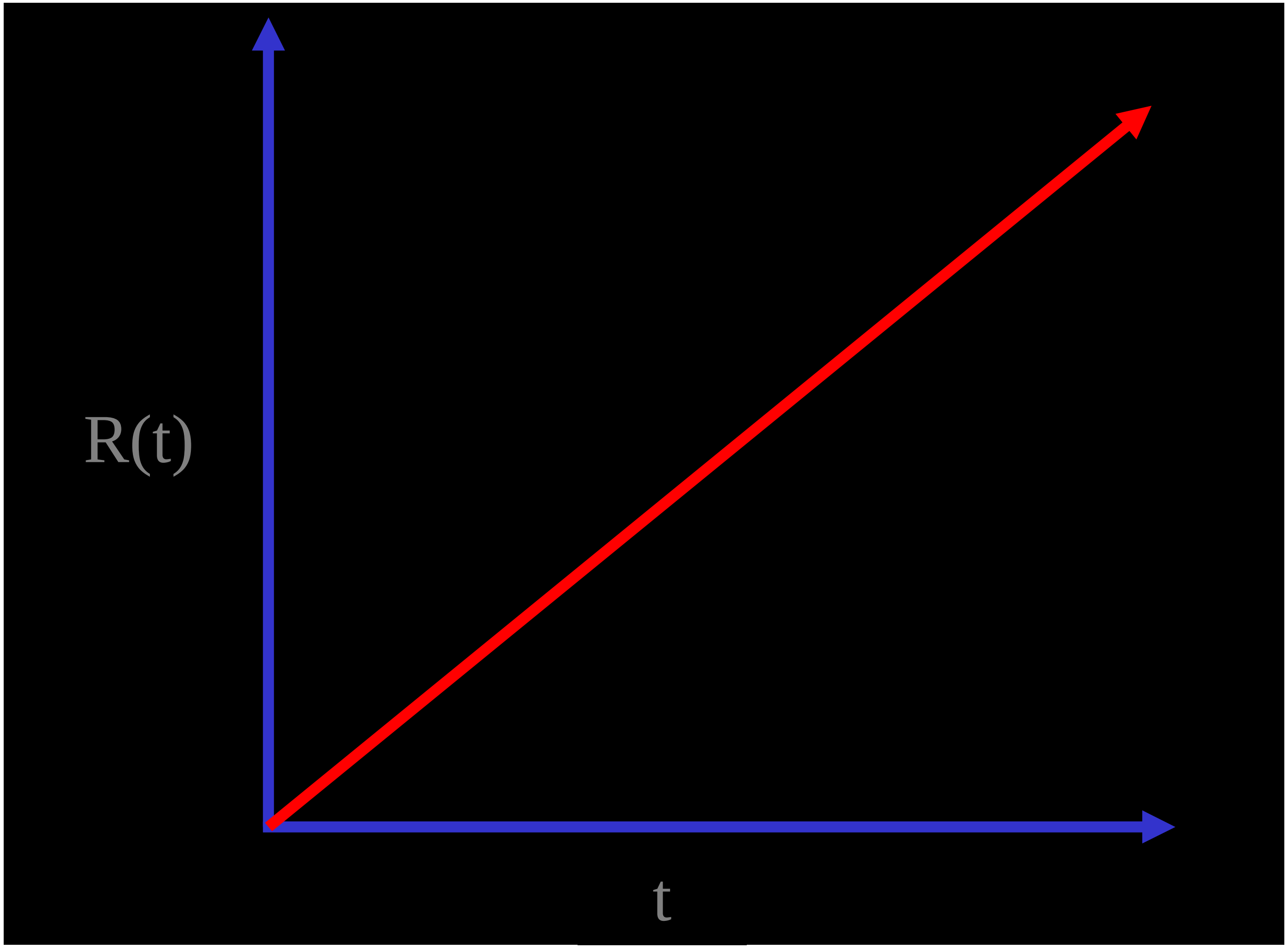
$\Rightarrow$ for cosmological redshift:

$$1 + z = \frac{\lambda_{received}}{\lambda_{emitted}} = \frac{R_{now}}{R_{then}}$$

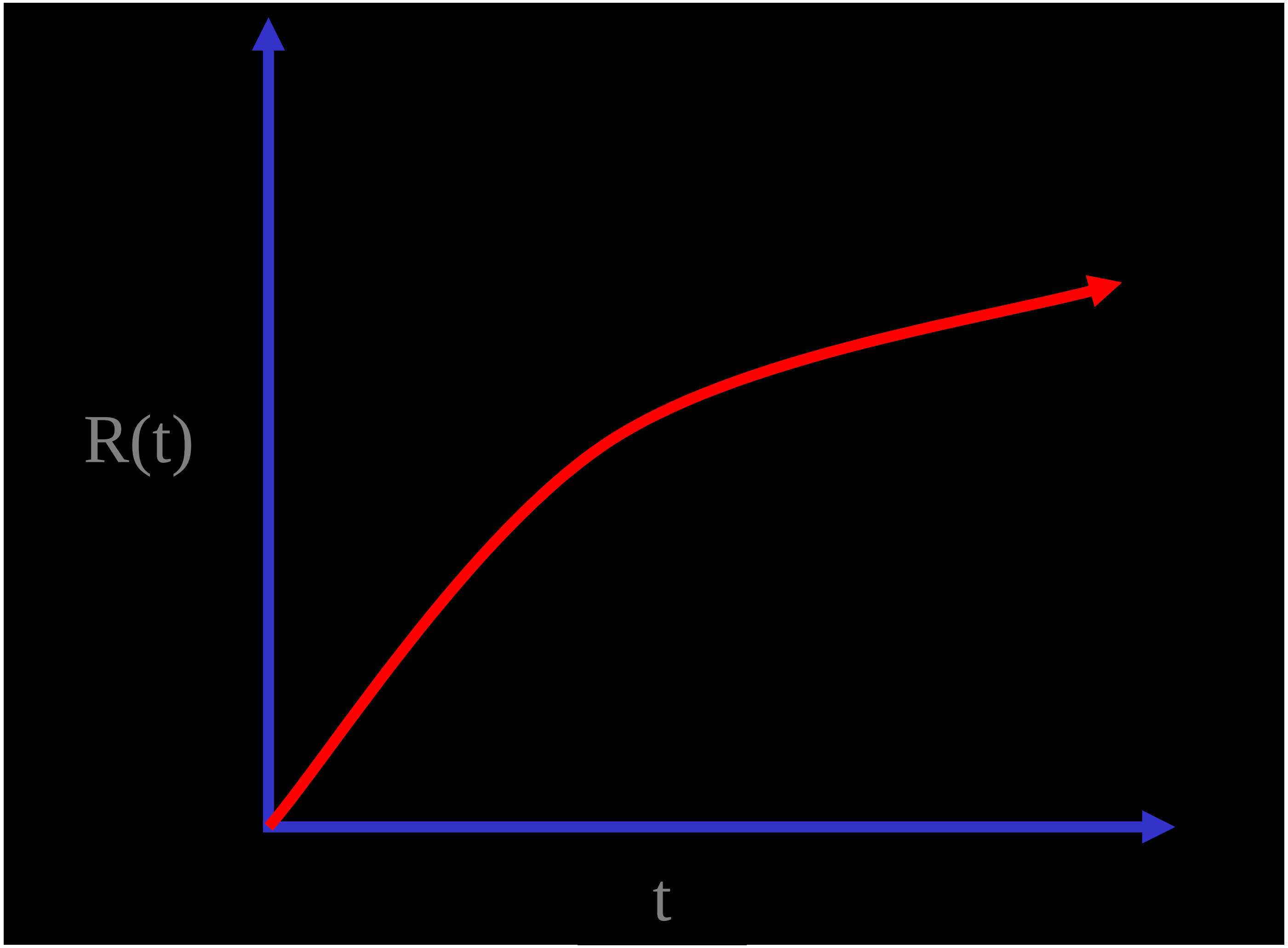
Example: static universe



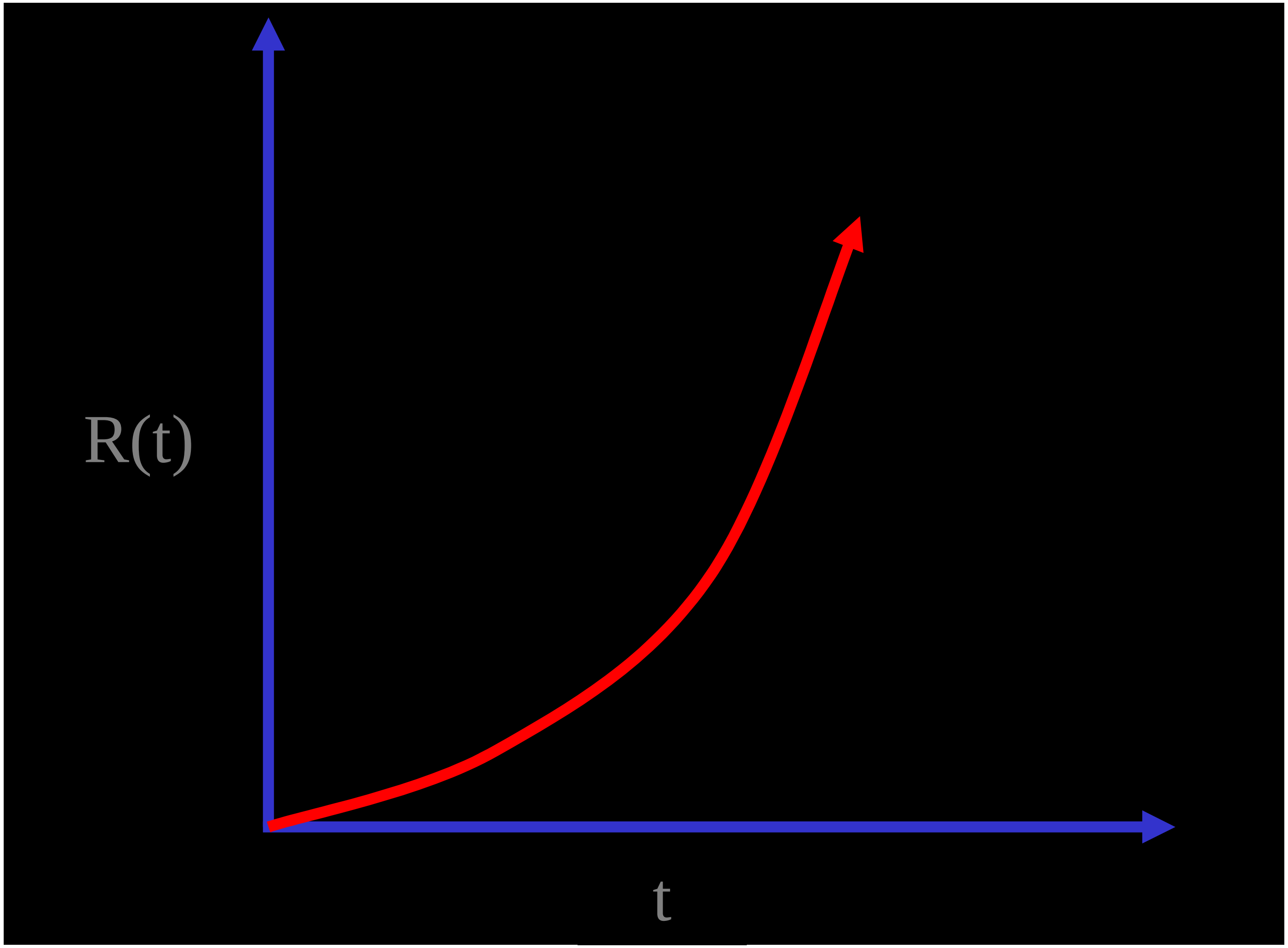
Example: expanding at a constant rate



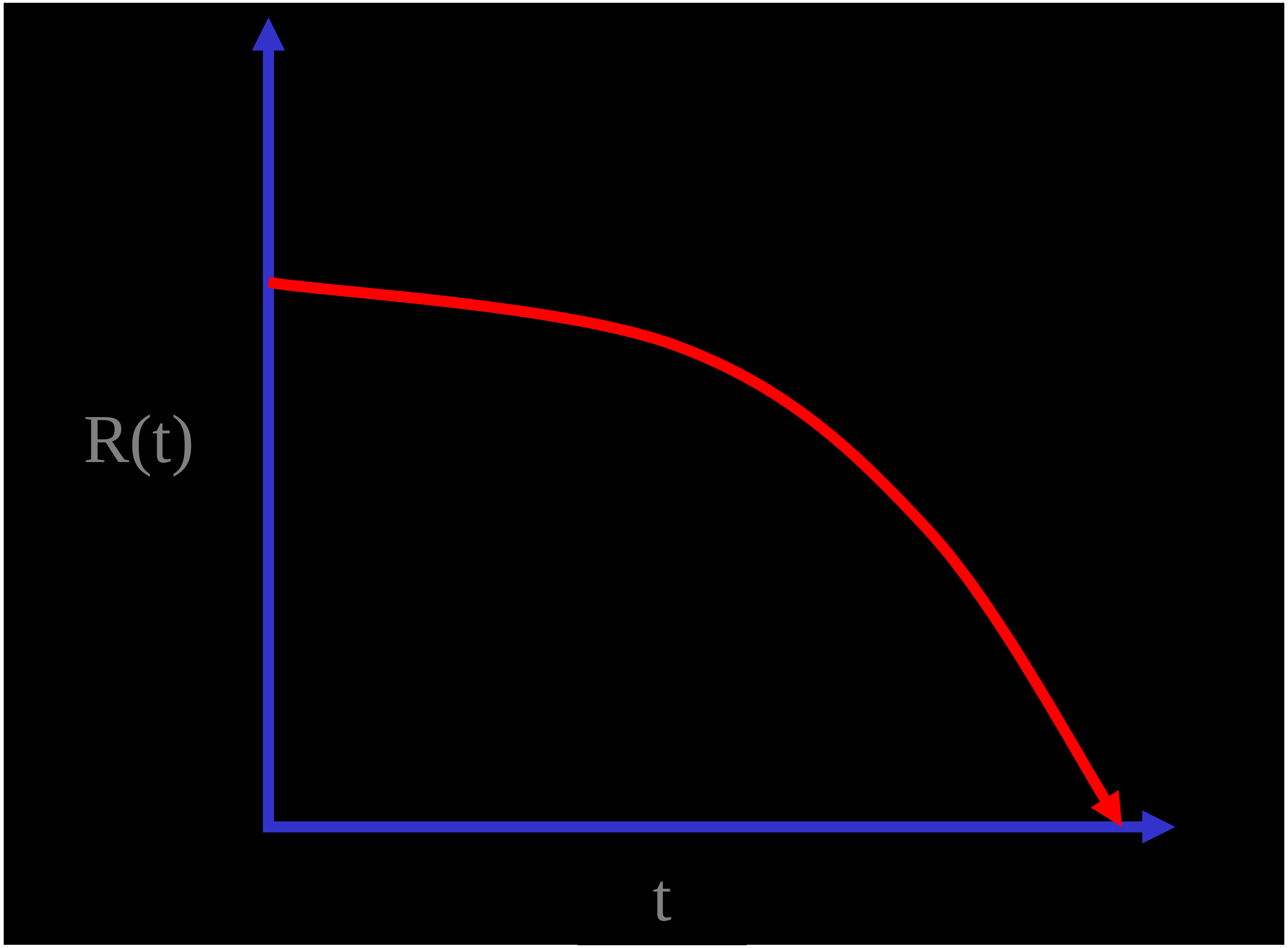
Example: expansion is slowing down



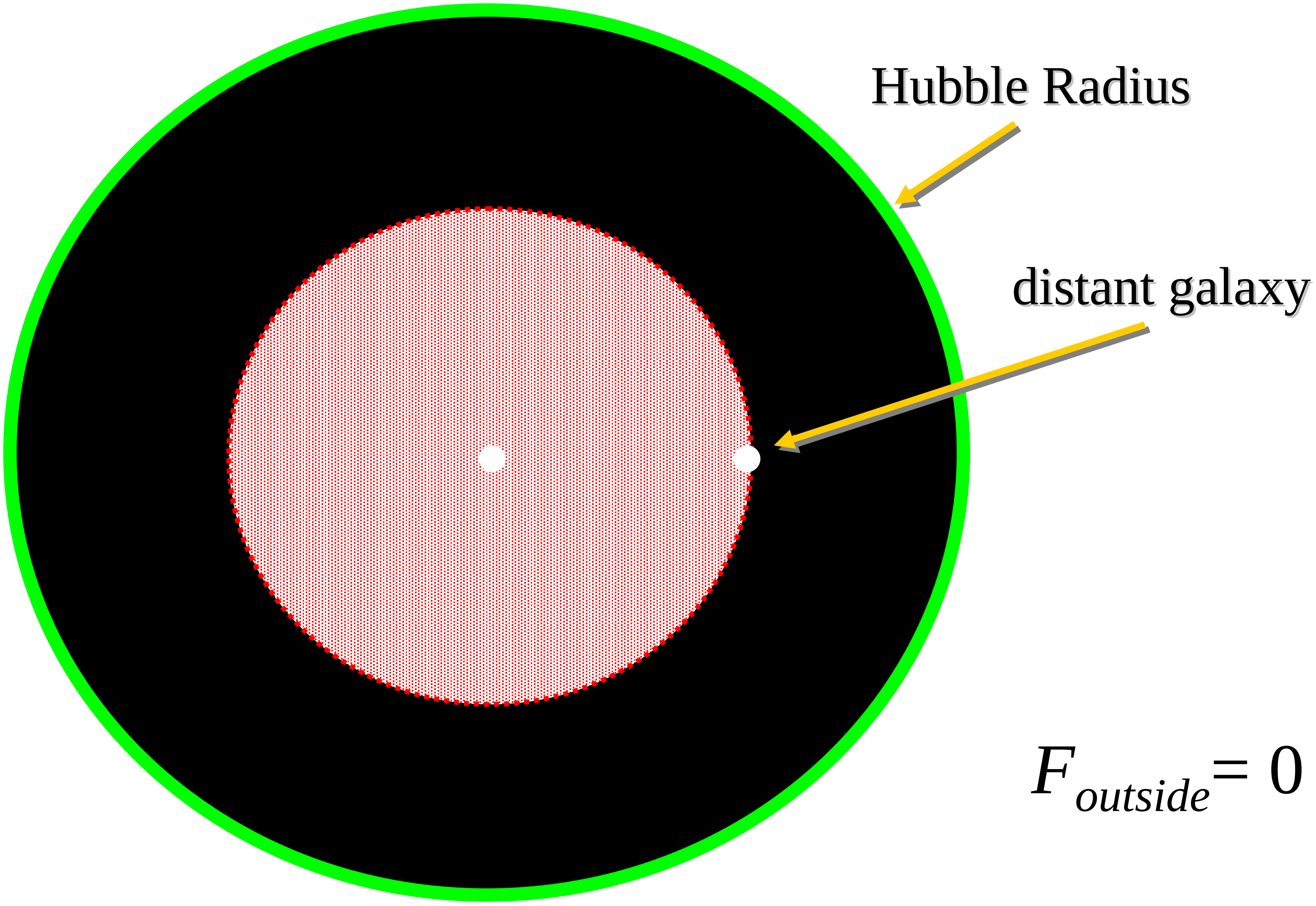
Example: expansion is accelerating



Example: collapsing

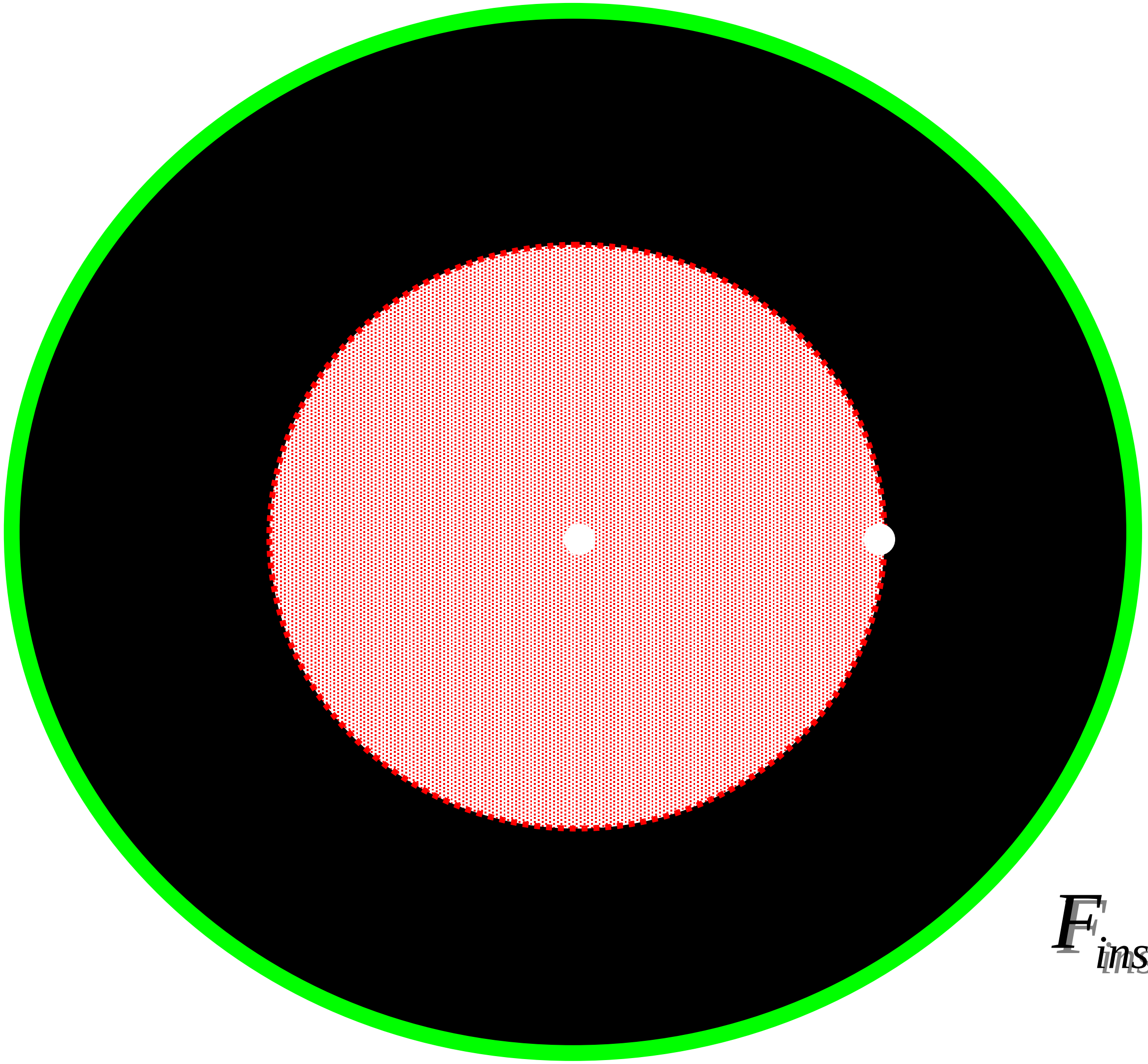


Can we calculate R(t) ?



$$F_{outside} = 0$$

Can we calculate R(t) ?



$$F_{inside} = -G\frac{M_{inside}m_{gal}}{R^2}$$

What is the future of that galaxy ?

- Critical velocity: escape speed

$$v_{esc} = \sqrt{\frac{2GM_{inside}}{R}}$$

- $v<v_{esc}$: galaxy eventually stops and falls back
- $v>v_{esc}$: galaxy will move away forever

Let's rewrite that a bit ...

$$v^2 = \frac{2GM_{inside}}{R} + 2\varepsilon_\infty$$

- $\varepsilon_\infty < 0 \Rightarrow v < v_{esc}$: galaxy eventually stops and falls back
- $\varepsilon_\infty > 0 \Rightarrow v > v_{esc}$: galaxy will move away forever

Let's rewrite that a bit ...

- Homogeneous sphere of density ρ :

$$M_{inside} = \frac{4\pi}{3}\rho\, R^3$$

- so for the velocity:

$$v^2 = \frac{8\pi G}{3}\rho R^2 + 2\varepsilon_\infty$$

- but what is ε_∞?

Let's switch to general relativity

- Friedmann equation (=Einstein eq. for ***homogeneous*** and ***isotropic*** mass/energy distribution!)

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

- same k as in the Robertson-Walker metric

$$ds^2 = (cdt)^2 - R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right)$$

Let's switch to general relativity

- Friedmann equation

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2$$

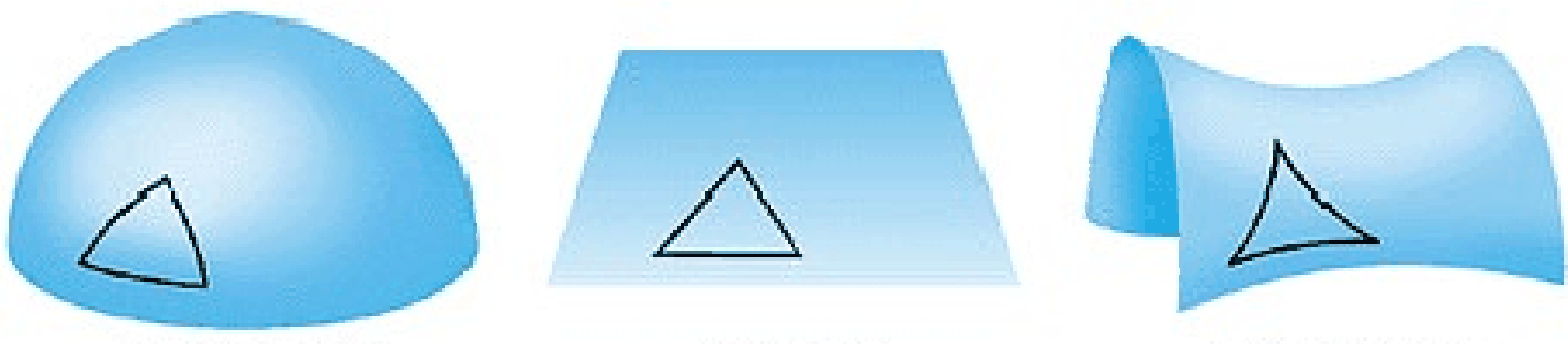
- k is the curvature constant

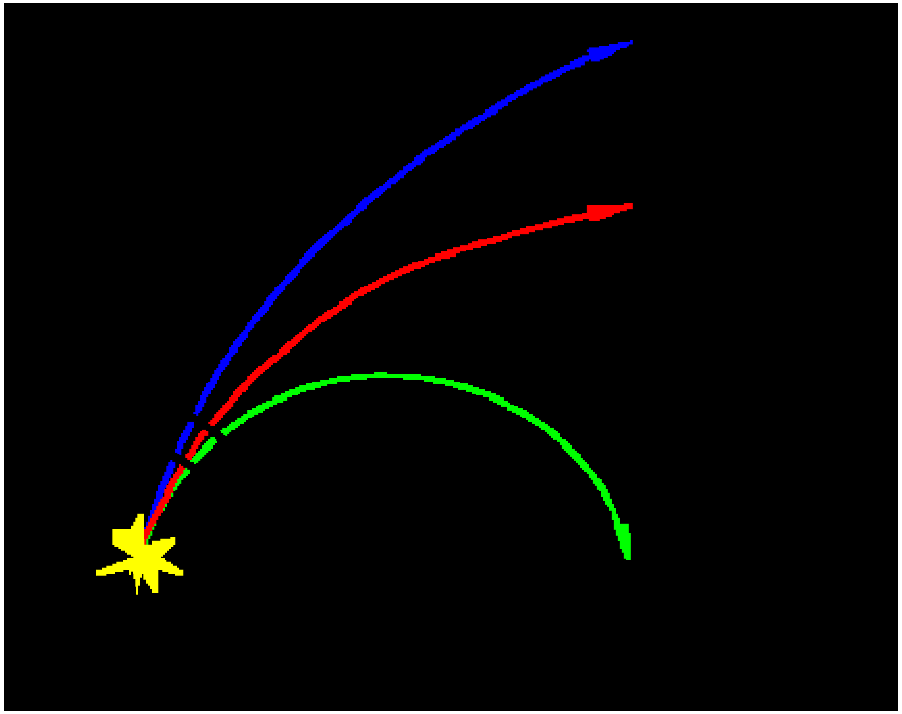
  $k=0$: flat space, forever expanding

  $k>0$: spherical geometry, eventually recollapsing

  $k<0$: hyperbolic geometry, forever expanding

$k>0$ $k=0$ $k<0$





Can we predict the fate of the Universe ?

- Friedmann equation:

$$H_0^2 = \frac{v^2}{R^2} = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}$$

- *k=0*:

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

Can we predict the fate of the Universe ?

- If the density ρ of the Universe
  - $\rho = \rho_{crit}$: flat space, forever expanding
  - $\rho > \rho_{crit}$: spherical geometry, recollapsing
  - $\rho < \rho_{crit}$: hyperbolic geometry, forever expanding
- so what is the density of the universe?
  - We don’t know precisely
  - $\rho > \rho_{crit}$ very unlikely
  - currently favored model: $\rho \approx \rho_{crit}$

How big is ρ_{crit} ?

- $\rho_{crit} = 8\times10^{-30}$ g/cm^3 $\approx$ 1 atom per 200 liter
- density parameter Ω_0

$$\Omega_0 = \frac{\rho}{\rho_{crit}} = \frac{3H_0^2\rho}{8\pi G}$$

$\Omega_0 = 1$: flat space, forever expanding (open)

$\Omega_0 > 1$: spherical geometry, recollapsing (closed)

$\Omega_0 < 1$: hyperbolic geometry, forever expanding

- currently favored model: $\Omega_0 \approx 1$

How can we measure Ω_0 ?

- Count all the mass we can “see”
  - tricky, some of the mass may be hidden …
- Measure the rate at which the expansion of the universe is slowing down
  - a more massive universe will slow down faster
- Measure the geometry of the universe
  - is it spherical, hyperbolic or flat ?

Let’s try to measure the deceleration

- Acceleration according to Newton:

$$a = -G\frac{M}{R^2} = -\frac{4\pi G\rho}{3}R$$

- deceleration parameter

$$q_0 = -\frac{aR}{v^2} = \frac{\Omega_0}{2}$$

So what's the meaning of q_0 ?

- deceleration parameter q_0

$q_0>0.5$:	deceleration is so strong that eventually the universe stops expanding and starts collapsing
$0<q_0<0.5$:	deceleration is too weak to stop expansion

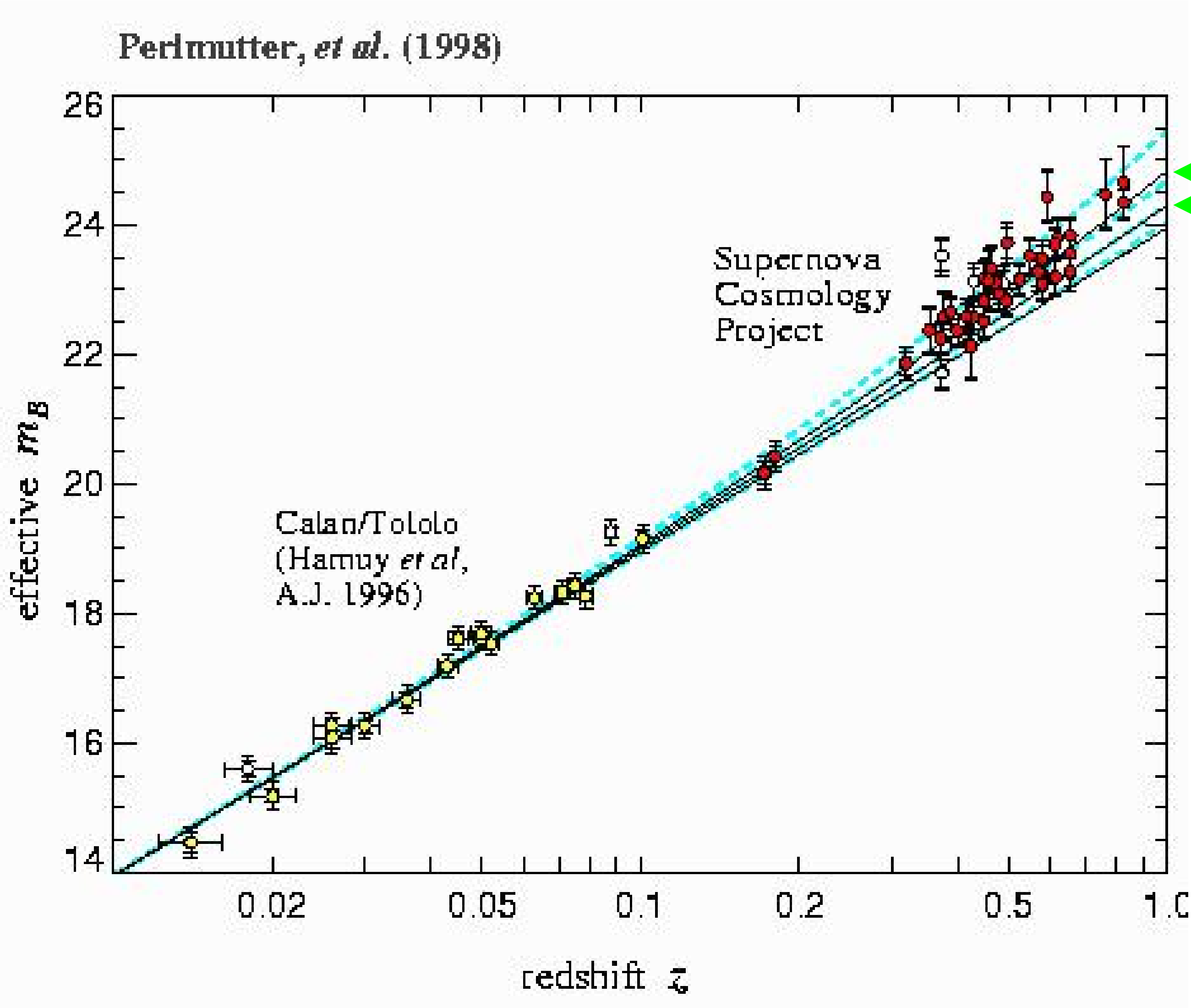
- What's the difference between q_0, Ω_0 and k ?

k:	curvature of the universe
Ω_0:	total energy content of the universe
q_0:	kinematics of the universe

So let's measure q_0 !

- How do we do that?
  - Measure the rate of expansion at different times, i.e. measure and compare the expansion based on nearby galaxies and based on high redshift galaxies
- Gravity is slowing down expansion $\Rightarrow$ expansion rate should be higher at high redshift.

So let's measure q_0 !



fainter

$q_0 = 0$

$q_0 = 0.5$

more distant

Data indicates:

$q_0 < 0$

$\Rightarrow$ Expansion is accelerating

Science discovery of the year 1998

- The expansion of the universe is **accelerating** !!!
- But gravity is always attractive, so it only can decelerate!

$\Rightarrow$ Revival of the cosmological constant Λ

Friedmann's equation for Λ>0

$$v^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 + \frac{\Lambda R^2}{3}$$

- k is the curvature constant

  $k=0$: flat space

  $k>0$: spherical geometry

  $k<0$: hyperbolic geometry

- but for sufficiently large Λ a spherically curved universe may expand forever

Deceleration parameter *q* for Λ>0

- Acceleration according to Newton:
- Acceleration according to Newton:

$$a = -\frac{4\pi G\rho}{3}R + \frac{\Lambda}{3}R$$

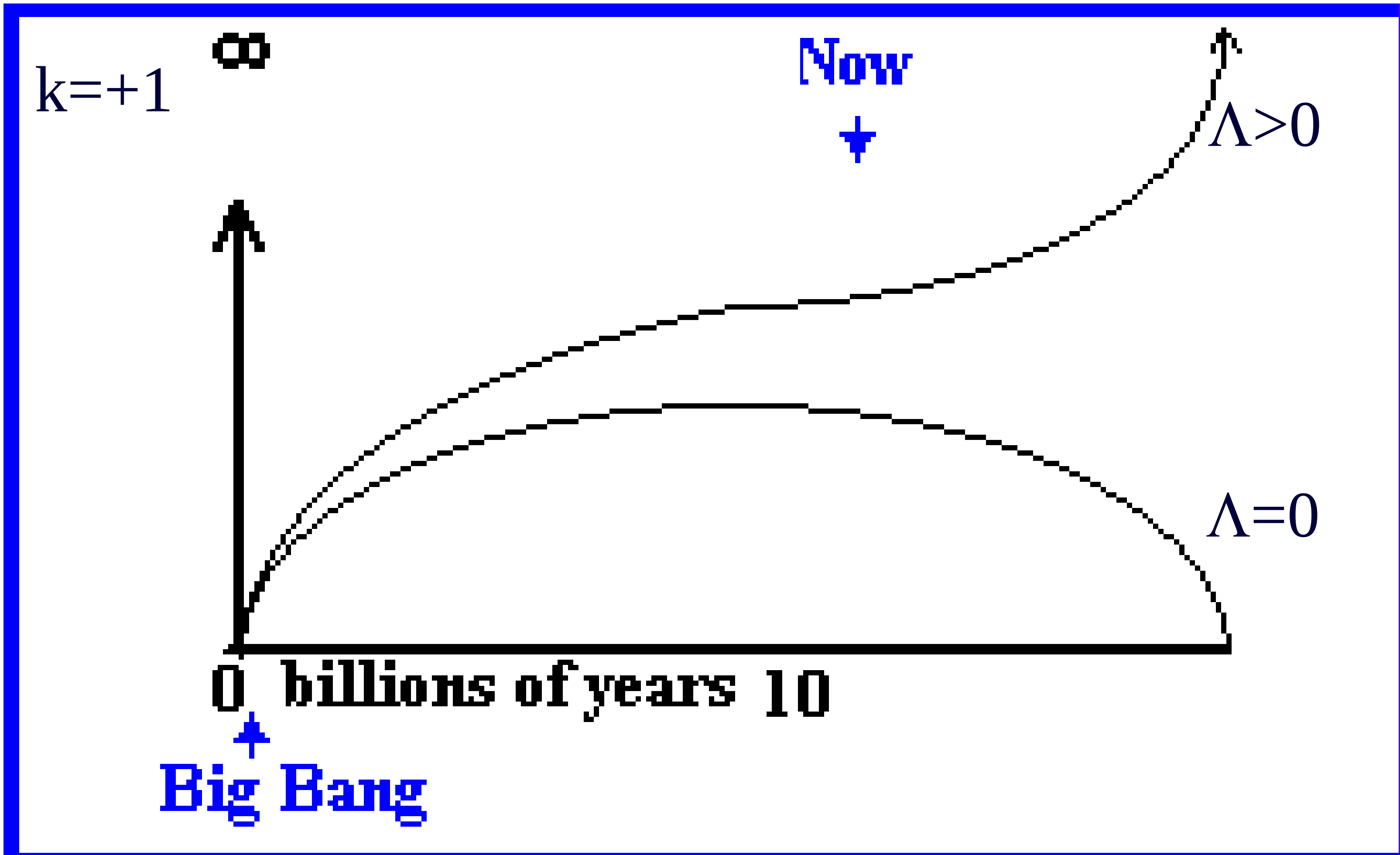
- deceleration parameter
- deceleration parameter

$$q_0 = -\frac{aR}{v^2} = \frac{\Omega_0}{2} - \Omega_\Lambda$$

with

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2}$$

The fate of the Universe for Λ>0



Is the fate of the Universe well determined ?

- deceleration:

  $\frac{1}{2}\Omega_m - \Omega_\Lambda > 0$: decelerating

  $\frac{1}{2}\Omega_m - \Omega_\Lambda < 0$: accelerating

- curvature

  $\Omega_m + \Omega_\Lambda = 1$: flat

  $\Omega_m + \Omega_\Lambda < 1$: hyperbolic

  $\Omega_m + \Omega_\Lambda > 1$: spherical

- two equations for two variables $\Rightarrow$ well posed problem

Cosmology: "the quest for three numbers"

- The Hubble constant H_0
  - $\Rightarrow$ how fast is the universe expanding
- The density parameter Ω_m
  - $\Rightarrow$ how much mass is in the universe
- The cosmological constant Ω_Λ
  - $\Rightarrow$ the vacuum energy of the universe
- current observational situation:
  - $H_0 = 73$ km/s/Mpc
  - $\Omega_m = 0.3$; $\Omega_\Lambda = 0.7 \Rightarrow$ flat space