Lecture 7   Compact Objects

Degenerate Objects

- In the leftover core of a dead star…
  - degeneracy pressure supports the star against the crush of gravity
- A degenerate star which is supported by:
  - electron degeneracy pressure is called a **white dwarf**
  - neutron degeneracy pressure is called a **neutron star**
- If the remnant core is so massive that the force of gravity is greater than neutron degeneracy pressure…
  - the star collapses out of existence and is called a **black hole**

Degenerate Core Leftover

- The central star of a Planetary Nebula heats up as it collapses.
- The star has insufficient mass to get hot enough to fuse Carbon.
- Gravity is finally stopped by the force of **electron degeneracy pressure**.
- The star is now stable…...

White Dwarf

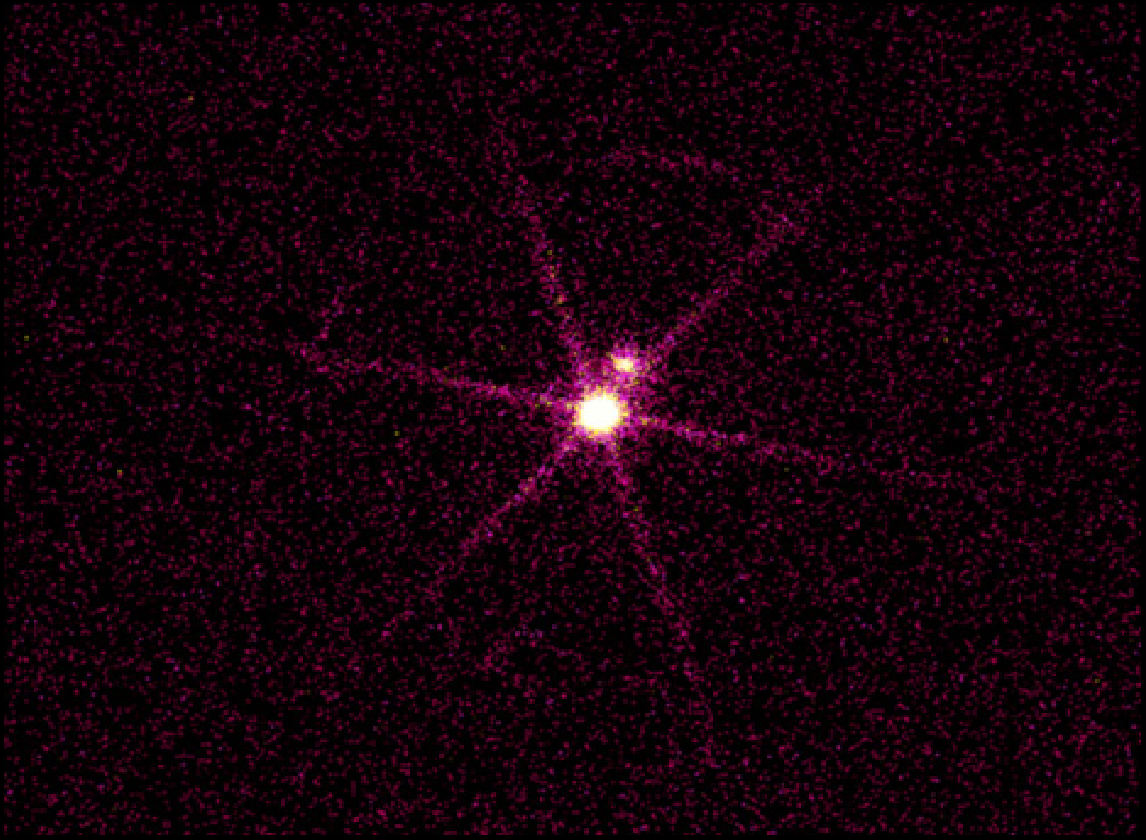
White Dwarf

White Dwarfs

- They are stable…
  - gravity vs. electron degeneracy *pressure*
- They generate no *new* energy.
- They slide down the HR-diagram as they radiate their heat into space, getting cooler and fainter.
- They are very dense; 0.5 - 1.4 $M_{\odot}$ packed into a sphere the size of the Earth!

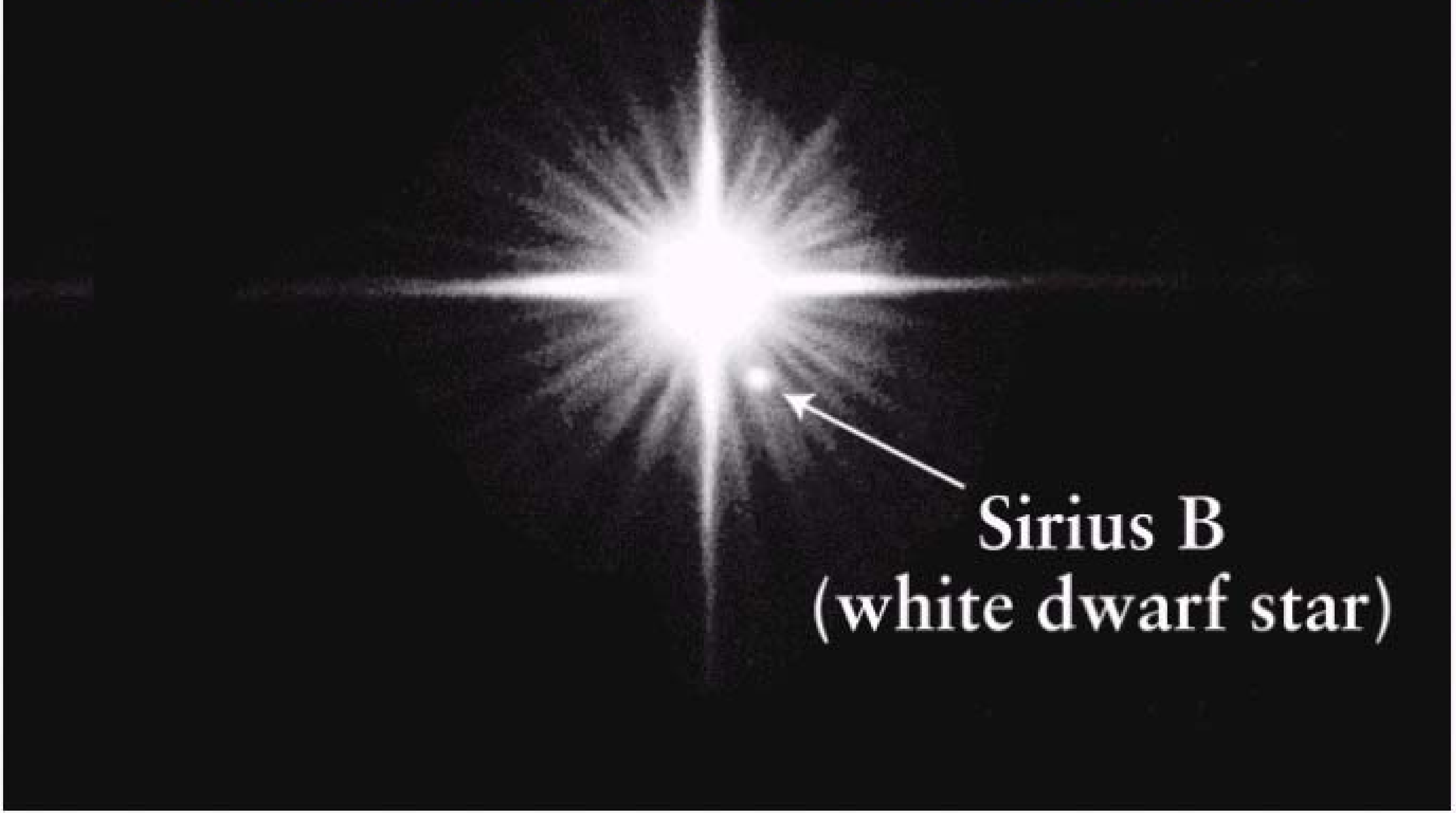
White Dwarfs

Sirius B is the closest white dwarf to us



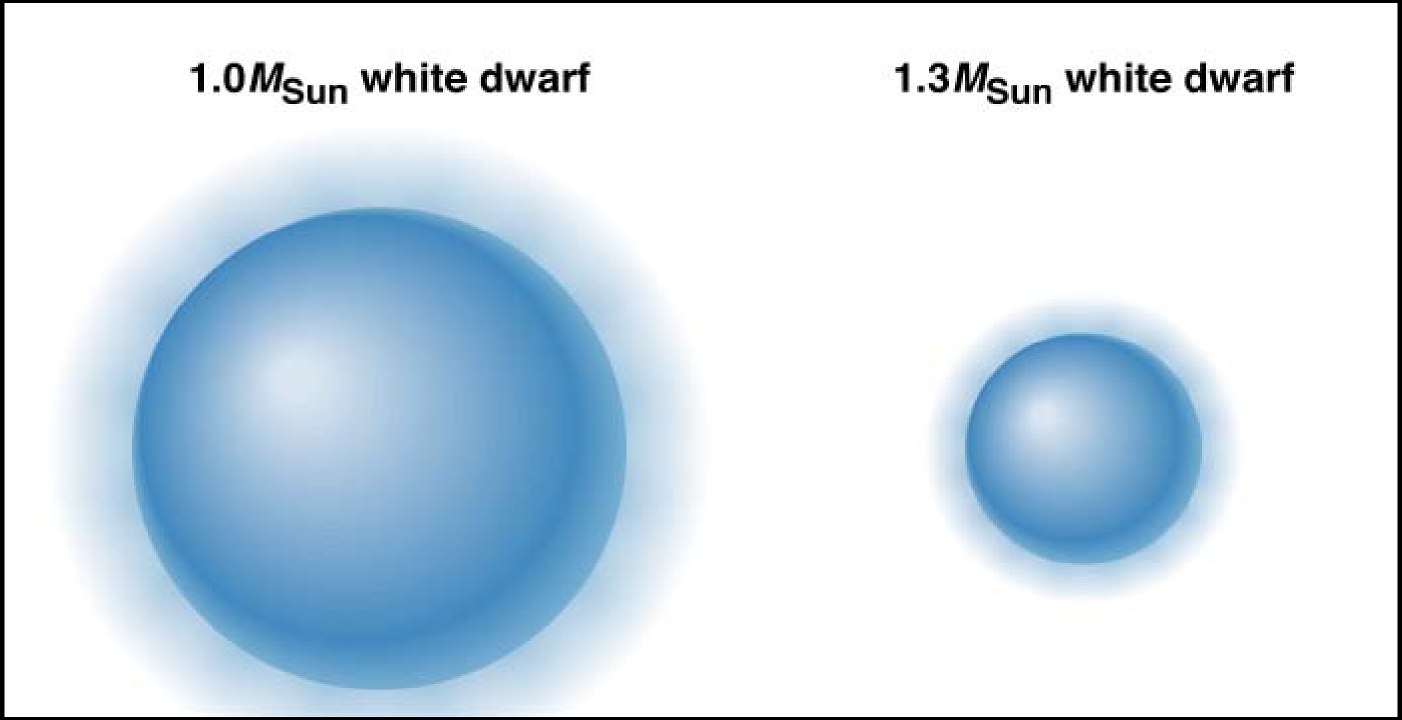
Sirius A + B in X-rays

The burned-out core of a low-mass star cools and contracts until it becomes a white dwarf.



White Dwarfs

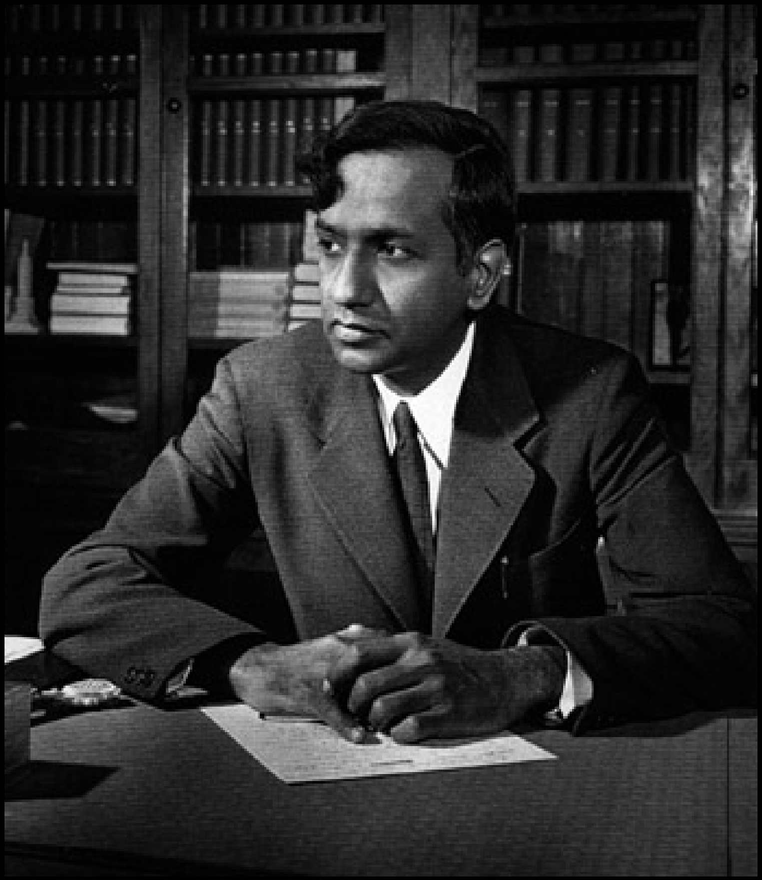
- Degenerate matter obeys unusual laws of physics.
- The more mass the star has, the *smaller* the star becomes!
  - increased gravity makes the star denser
  - greater density increases degeneracy pressure to balance gravity



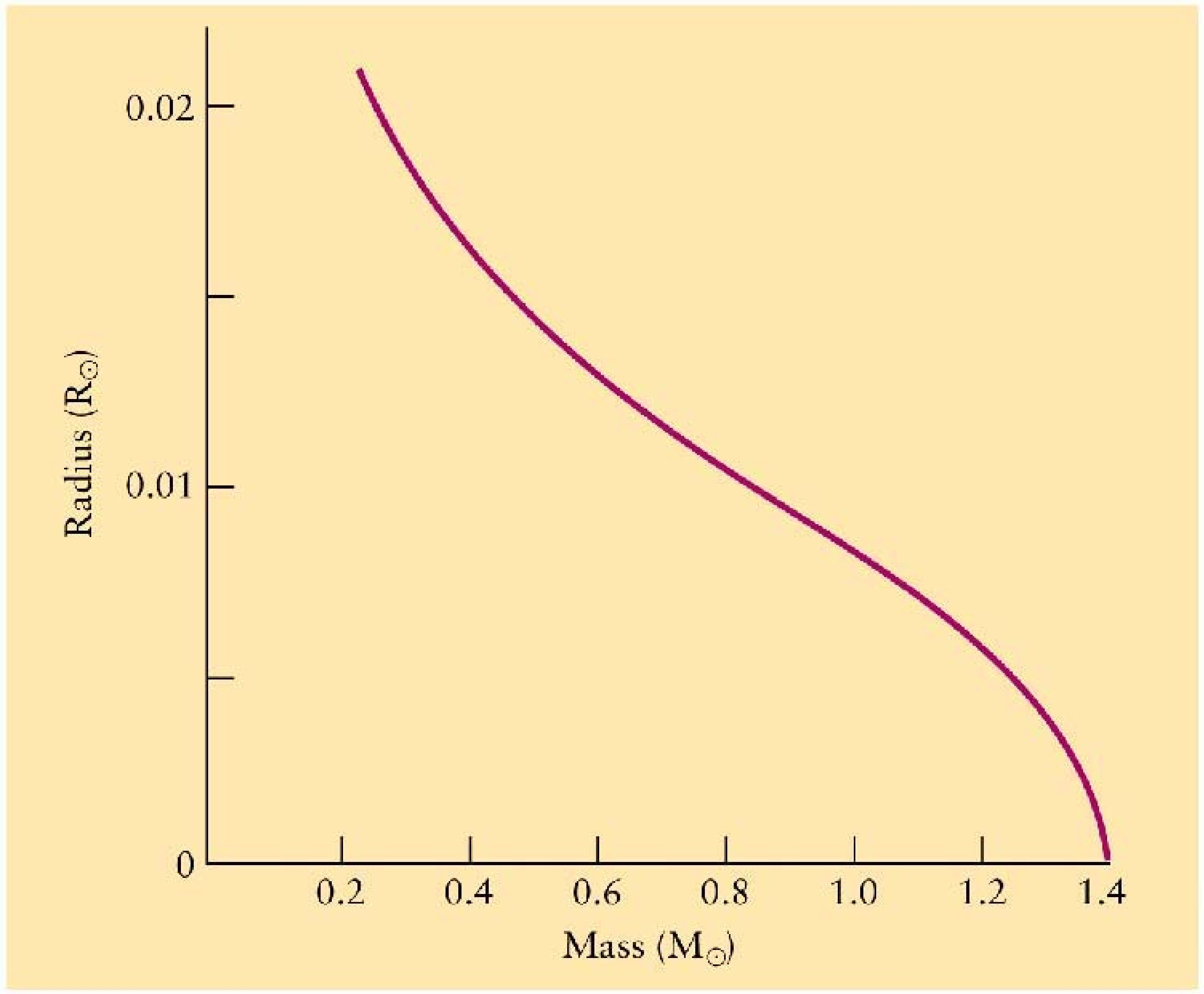
Limit on White Dwarf Mass

- Chandra formulated the laws of degenerate matter.
  - for this he won the Nobel Prize in Physics
- He also predicted that gravity will overcome the pressure of electron degeneracy if a white dwarf has a mass > $\boxed{1.4\ M_\odot}$
  - energetic electrons, which cause this pressure, almost reach the speed of light

Chandrasekhar Limit



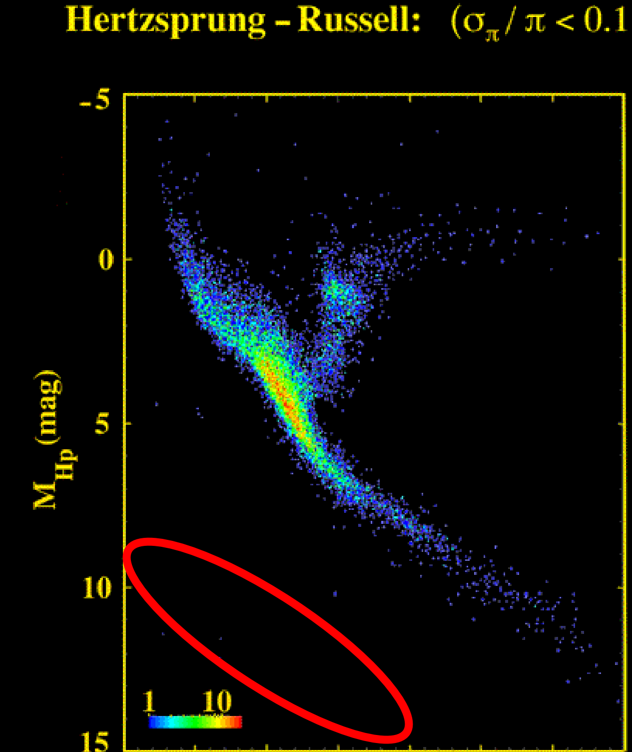
Subrahmanyan Chandrasekhar
(1910-1995)

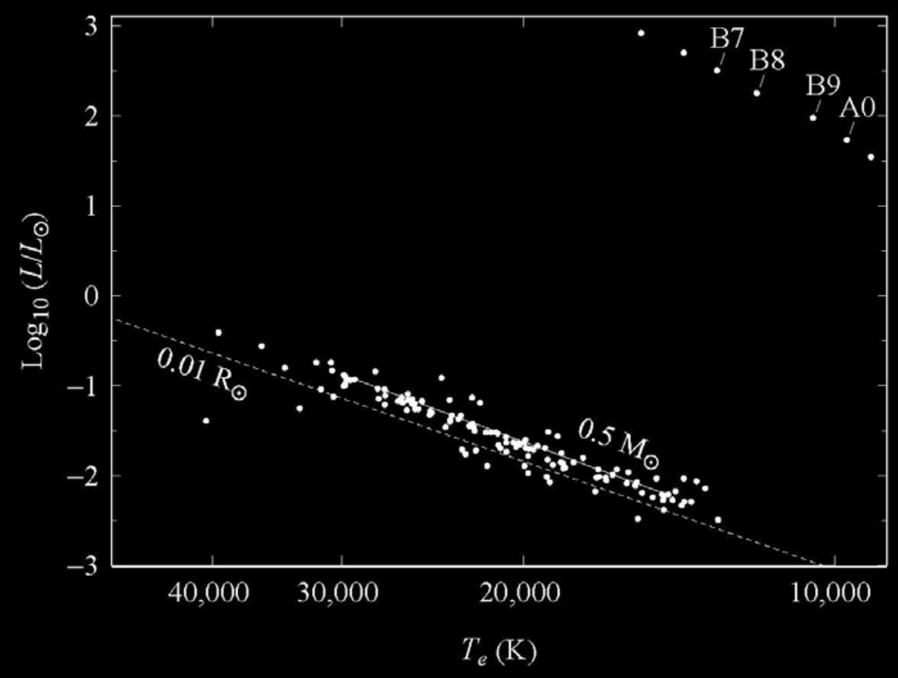


Radius ($R_\odot$)
0.02
0.01
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Mass ($M_\odot$)

White dwarfs

- Although they should be very common (as the remnant of normal stellar evolution) they are very hard to detect
- Though the hottest ones really are white, they have a range of temperatures and therefore colours.
- The central temperature of Sirius B is estimated to be ~7.6×10^7 K.
  - Clearly the low luminosity does not arise from hydrogen fusion.





Degenerate matter

- ***Pauli exclusion principle***: at most one fermion can occupy any given quantum state.
- The *Fermi energy* is the energy that divides occupied and unoccupied states at 0K.

$$\varepsilon_F = \frac{\hbar^2}{2m_e}\left[3\pi^2\left(\frac{Z}{A}\right)\frac{\rho}{\mu m_H}\right]^{2/3}$$

- Roughly speaking, if the thermal energy is less than the Fermi energy, an electron cannot make a transition to an unoccupied state.
  - i.e. the state of the electron is determined by degeneracy and not its thermal energy

- The electron degeneracy pressure is derived from the Pauli exclusion principle and the Heisenberg uncertainty principle:

$$P = \frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

Mass-Volume Relation

- Calculate the relationship between mass and volume for a completely degenerate star of constant density.

$$MV = \text{constant}$$

- More massive stars are *smaller*. Electrons must be more closely packed in more massive stars, for degeneracy to provide sufficient pressure.

- Clearly a problem here because if you keep piling mass on it's volume must go to zero. The derivation ignored relativity, and at high enough densities the velocities of the electrons approach the speed of light.

Chandrasekhar limit

- The velocities of the electrons are actually smaller than predicted by ignoring relativity. Thus they contribute less pressure: the volume will be even smaller than predicted earlier.
  - In fact, volume goes to zero for a finite mass.
  - *There is a maximum mass that a white dwarf can have.*

- The relativistic expression for pressure is:

$$P = \frac{\left(3\pi^2\right)^{1/3}}{4}\hbar c\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{4/3}$$

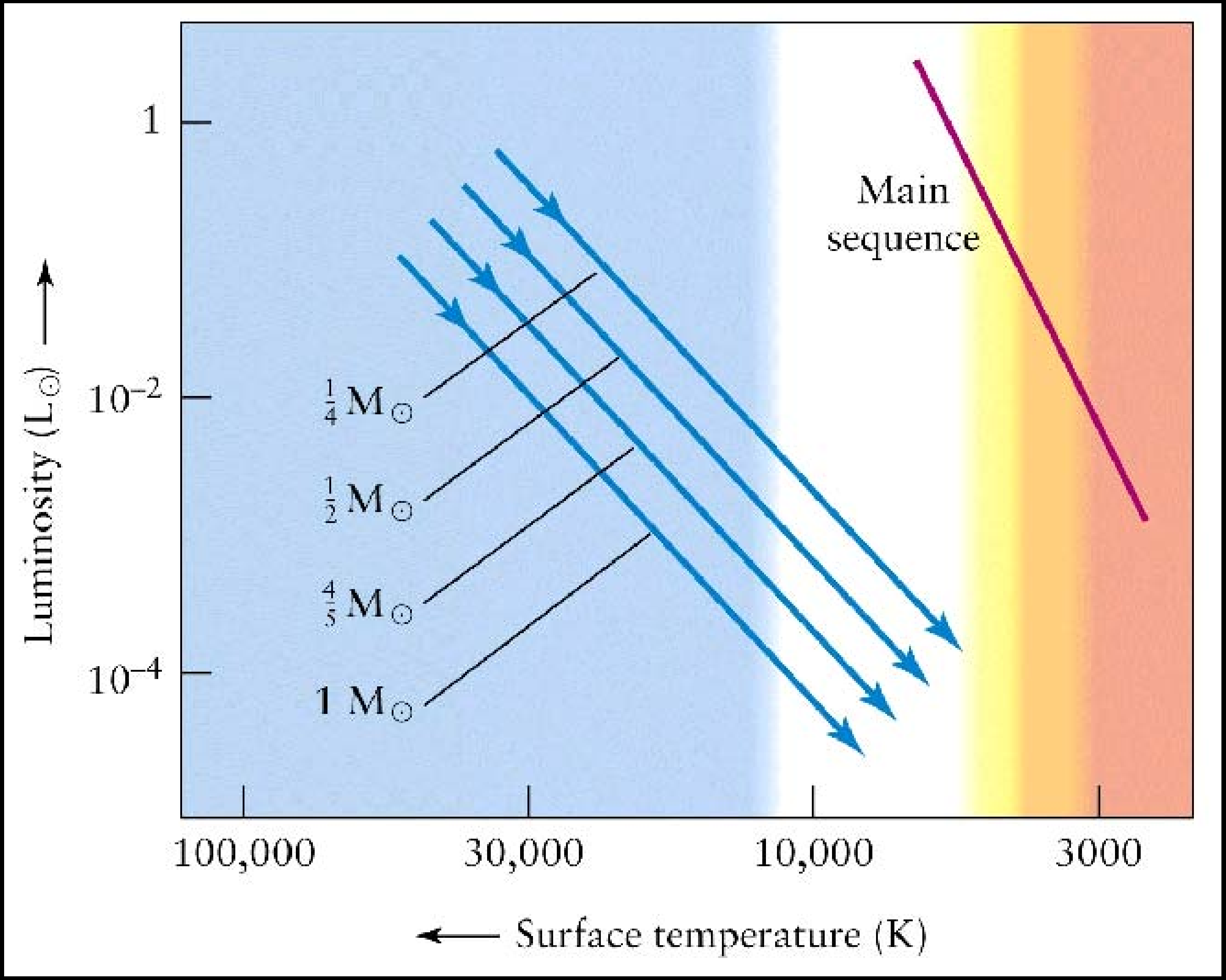
- Which leads to the *Chandrasekhar mass limit*:

$$M_{Ch} \approx \frac{3\sqrt{2\pi}}{8}\left(\frac{\hbar c}{G}\right)^{3/2}\left[\left(\frac{Z}{A}\right)\frac{1}{m_H}\right]^2$$

- (contains elements of quantum mechanics, relativity, and gravity!)

- A more careful calculation shows: $M_{Ch} \approx 1.44 M_{Sun}$

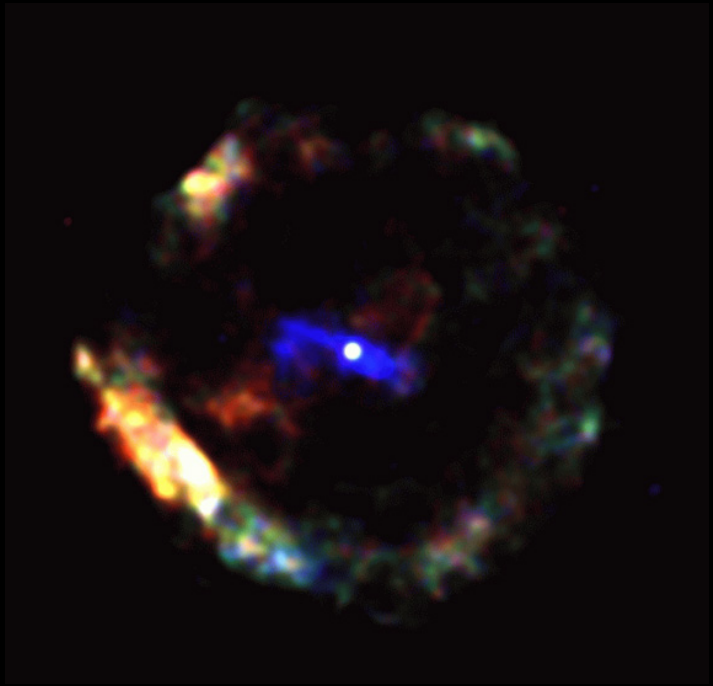
White dwarf cool curves



Neutron Stars

- …are the leftover cores from supernova explosions.
- If the core < 3 $M_\odot$, it will stop collapsing and be held up by neutron degeneracy pressure.
- Neutron stars are very dense (10^{12} g/cm^3)
  - 1.5 $M_\odot$ with a diameter of 10 to 20 km
- They rotate very rapidly: Period = milliseconds to 4 sec
- Their magnetic fields are 10^{13} times stronger than Earth's.

Chandra X-ray image of the neutron star left behind by a supernova observed in A.D. 386. The remnant is known as G11.2–0.3.



Neutron stars

- If a white dwarf exceeds the Chandrasekhar mass limit ($1.4M_{Sun}$) it must collapse until neutron degeneracy pressure takes over.

- A neutron star with a mass at the Chandrasekhar limit has a radius of only 10-15 km, and consists of $\sim 10^{57}$ neutrons: essentially it is an enormous nucleus.

$$M \approx 1.4 M_{Sun}$$

$$R \approx 10 km$$

$$\bar{\rho} \approx 6.65 \times 10^{17} kg / m^3 \approx 2.9 \rho_{nuclear}$$

Neutron stars

$$M \approx 1.4 M_{Sun}$$
$$R \approx 10 km$$
$$\overline{\rho} \approx 6.65 \times 10^{17} kg/m^3 \approx 2.9 \rho_{nuclear}$$

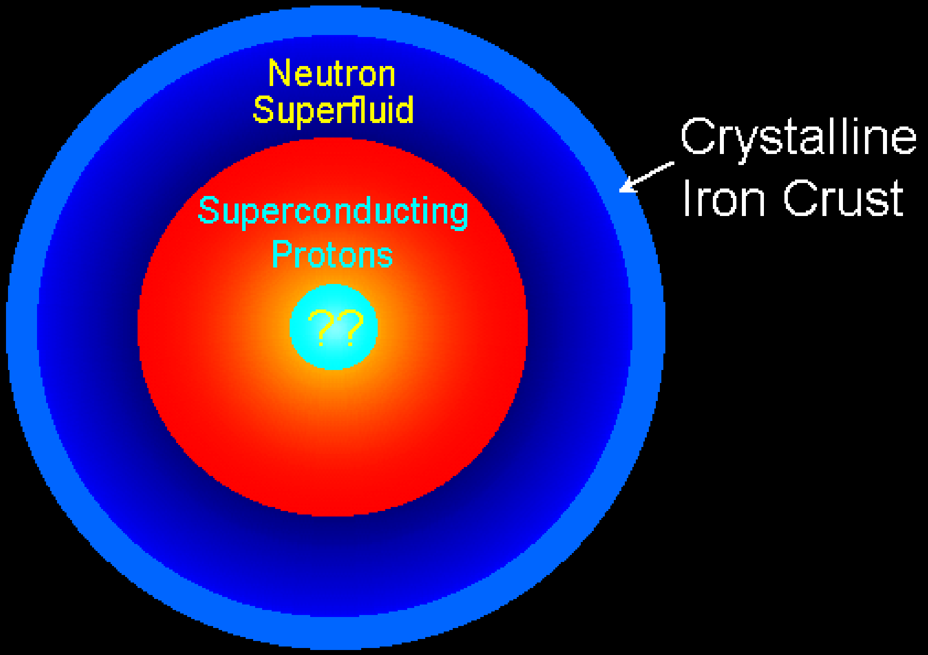
The force of gravity at the surface is very strong:

$$g = \frac{GM}{R^2} \approx 1.8 \times 10^{12} m/s^2$$

- An object dropped from a height of 1 m would hit the surface at a velocity 0.6% the speed of light.
- Must use general relativity to model correctly

Neutron stars: structure

1. **Outer crust:** heavy nuclei in a fluid ocean or solid lattice. Fe-56 is nearest the surface, with more neutron-rich species appearing at higher densities.
2. **Inner crust:** a mixture of neutron-rich nuclei, superfluid free neutrons and relativistic electrons.
3. **Interior**: primarily superfluid neutrons, with a few superfluid, superconducting protons and relativistic, degenerate electrons
4. **Core**: conditions here, at densities much larger than that of a typical nucleus, are uncertain. Likely consist of pions and other elementary particles. May even consist of quark-gluon plasma.



Neutron stars: rotation

- Conservation of angular momentum led to the prediction that neutron stars must be rotating very rapidly.

$$L = I\omega \propto MR^2\omega$$

$$\frac{\omega_i}{\omega_f} = \frac{P_f}{P_i} = \left(\frac{R_f}{R_i}\right)^2$$

- Neutron stars are ~500 times smaller than white dwarfs of the same mass, so their rotational periods are a few hundred thousand times shorter.
- Typical (observed) rotation periods for white dwarfs are ~20 minutes. Thus neutron stars must have periods of a few milliseconds.

Neutron stars: luminosity

- What is the blackbody luminosity of a 1.4 M_{Sun} neutron star, with a surface temperature of 1 million K?

$$
\begin{aligned}
L &= 4\pi\sigma R^2 T_e^4 \\
&= 7.1\times10^{25} W \\
&= 0.2 L_{Sun}
\end{aligned}
$$

- That's pretty bright. But, at what wavelength is it emitted?

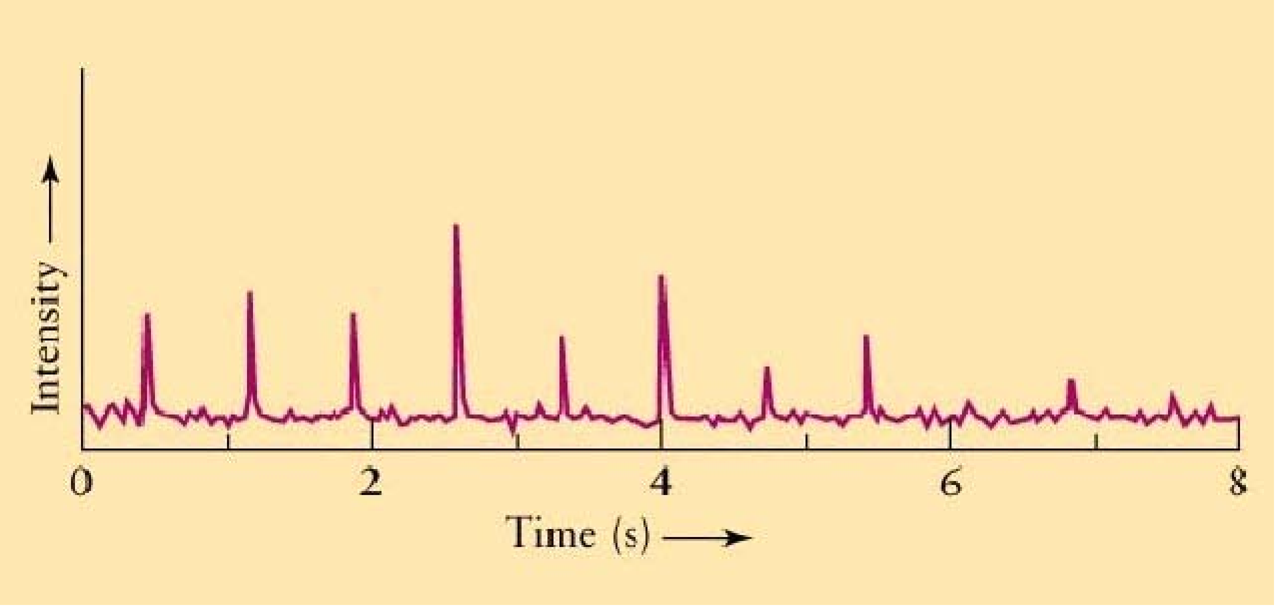
$$
\lambda_{\max} \propto \frac{1}{T}
$$

$$
\lambda_{\max} = \frac{(500nm)(5800K)}{T} = 2.9nm
$$

- This is at X-ray wavelengths and difficult to detect.

The discovery of pulsars in the 1960s stimulated interest in neutron stars.

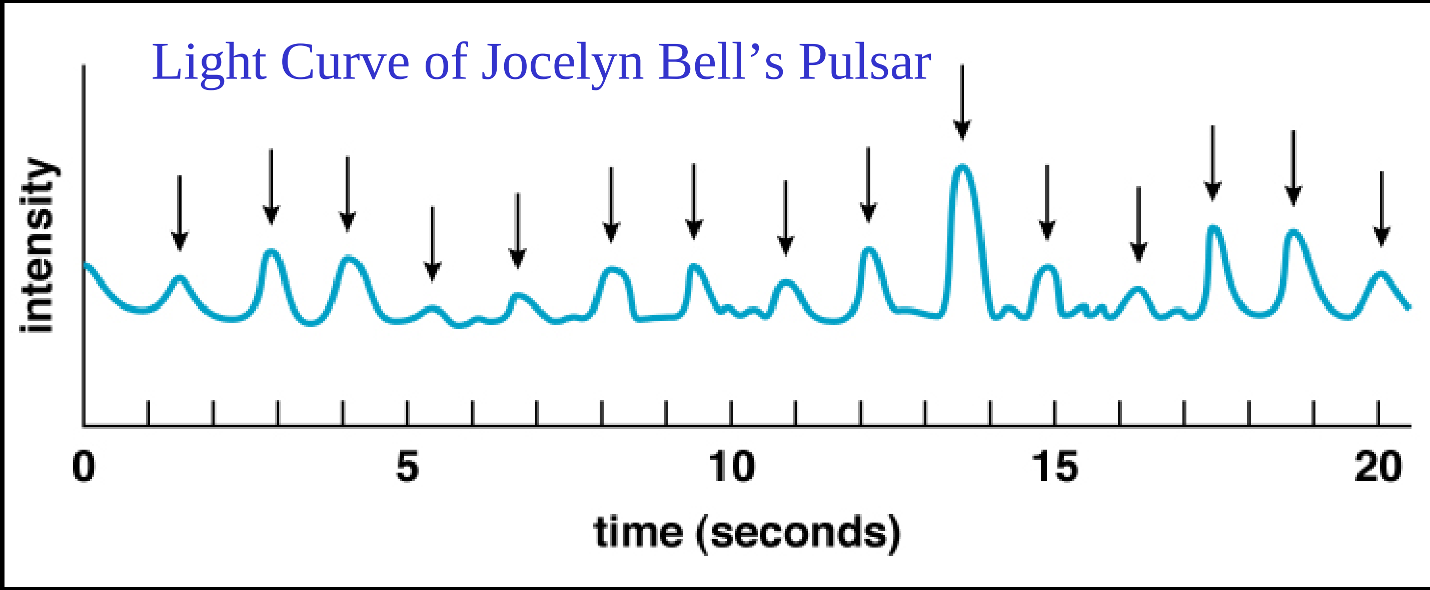
- First detected in 1967 by Cambridge University graduate student Jocelyn Bell.
- Radio source with an regular on-off-on cycle of exactly 1.3373011 seconds.



Pulsars

- In 1967, graduate student Jocelyn Bell and her advisor Anthony Hewish accidentally discovered a radio source in *Vulpecula*.
- It was a sharp pulse which recurred every 1.3 sec.
- They determined it was 300 pc away.
- They called it a **pulsar**, but what was it?

Jocelyn Bell



The discovery of pulsars in the 1960s stimulated interest in neutron stars.

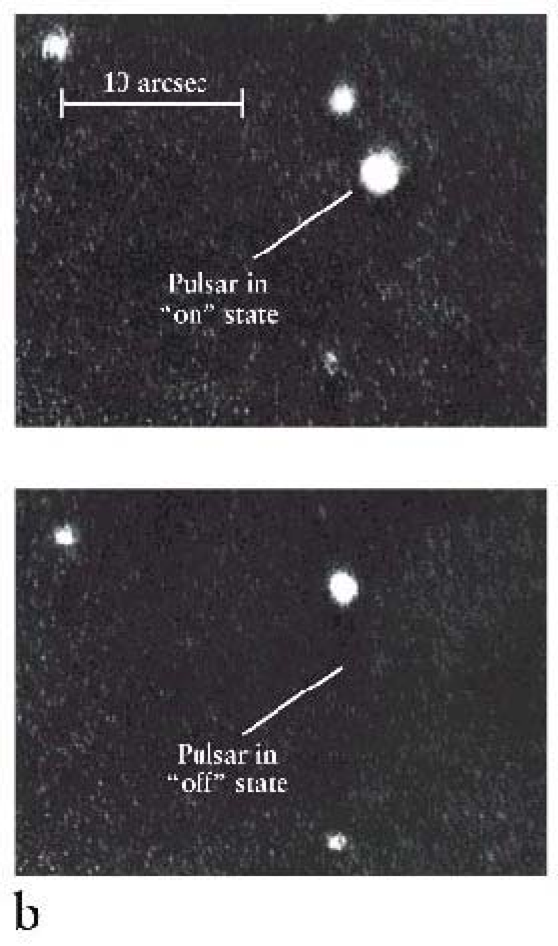
- First detected in 1967 by Cambridge University graduate student Jocelyn Bell.
- Radio source with an regular on-off-on cycle of exactly 1.3373011 seconds.
- Some scientists speculated that this was evidence of an alien civilization's communication system and dubbed the source LGM:

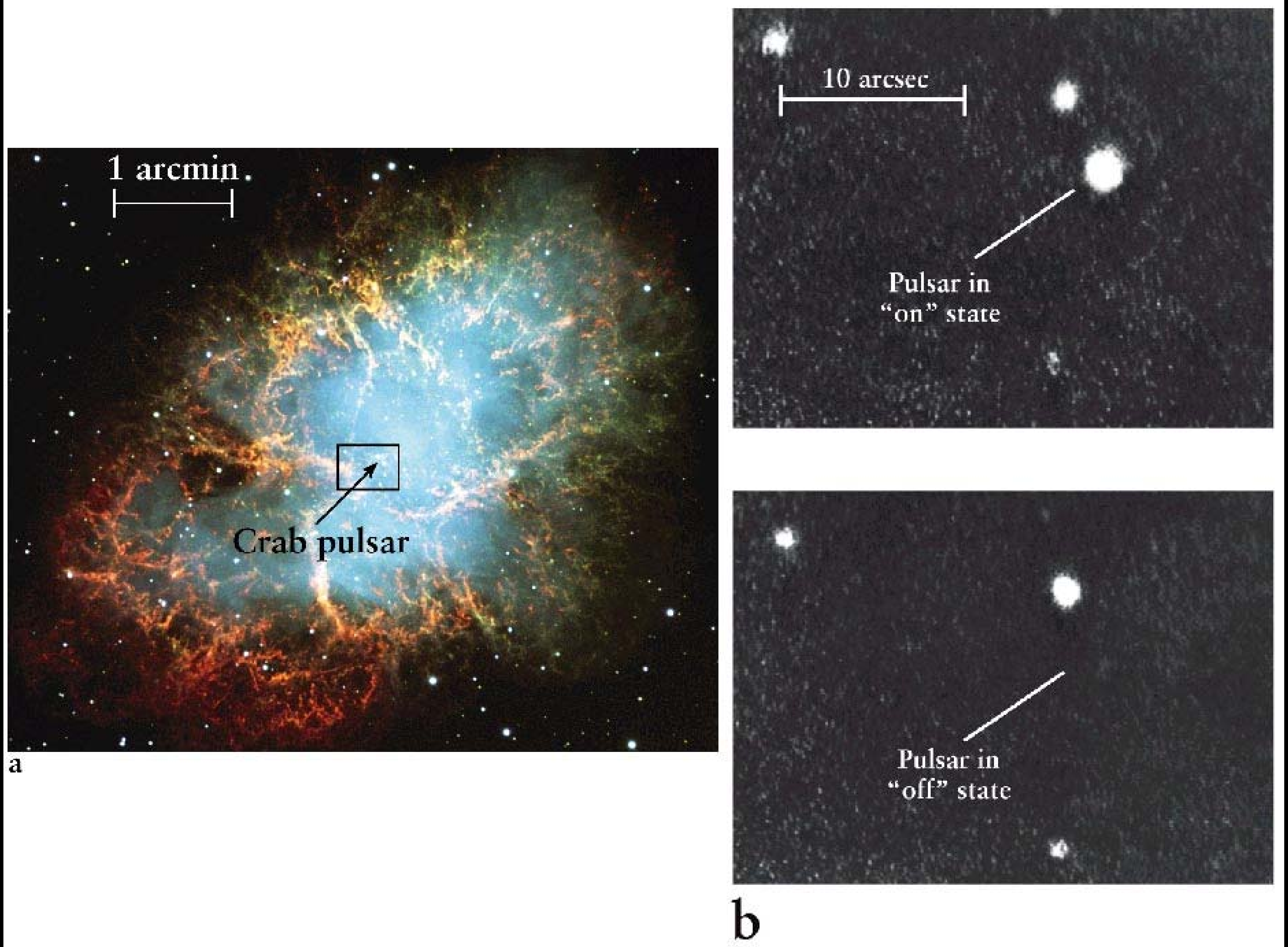
Little Green Men

- Today, we think pulsars are rapidly spinning neutron stars.

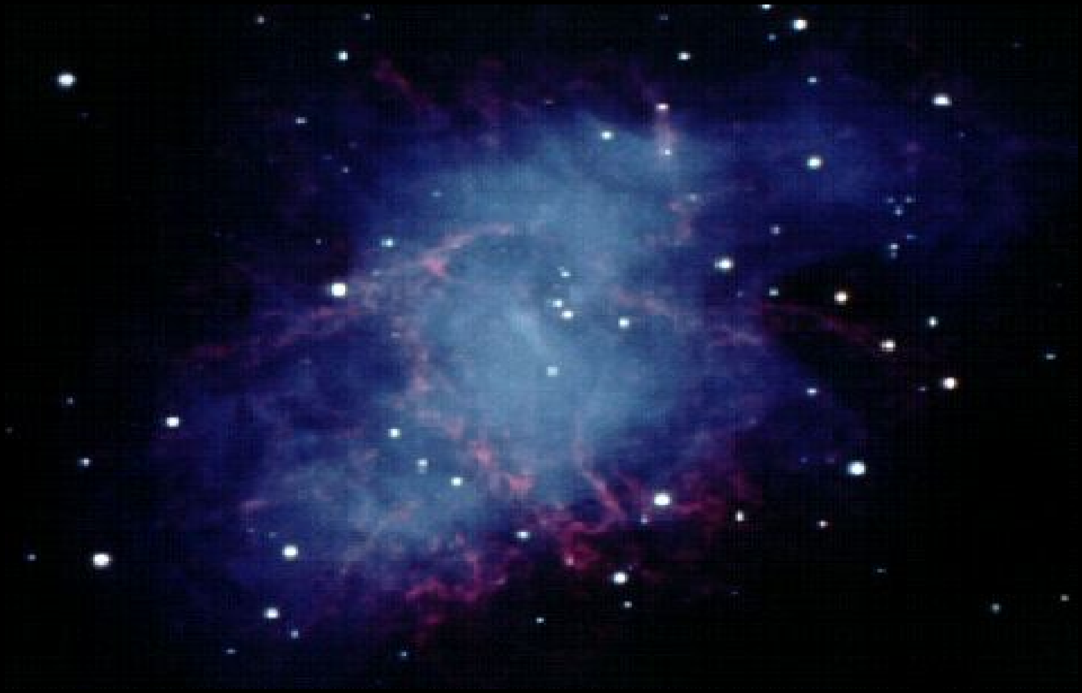
Pulsars are rapidly rotating neutron stars with intense magnetic fields.

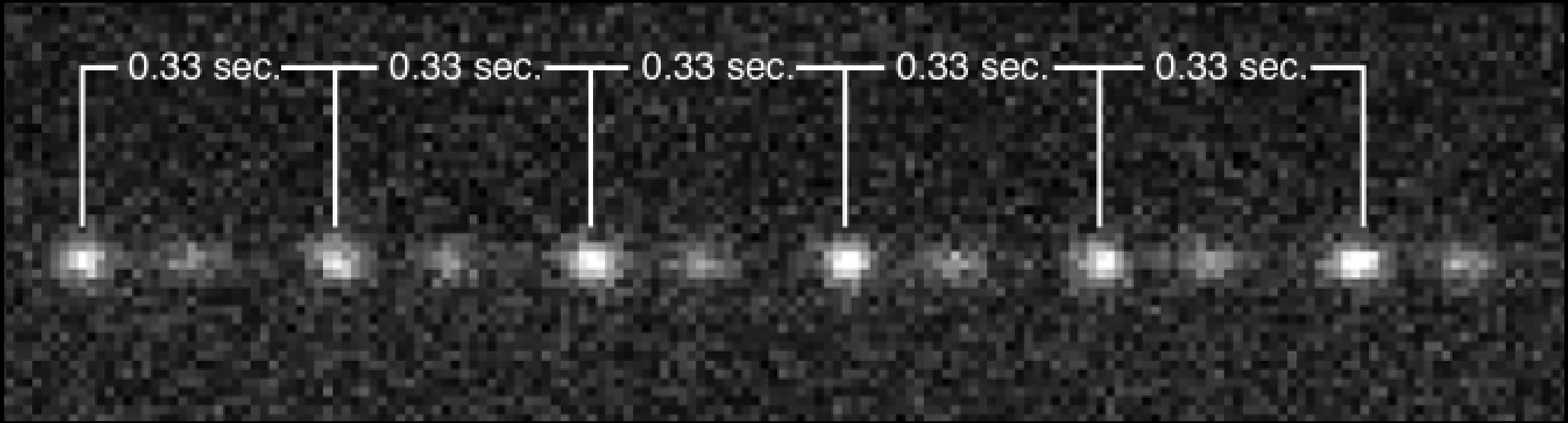
- Early ideas about pulsars were that there were pulsating white dwarfs; however, even a white dwarf (Earth-sized) is too big to oscillate in less than one second.
- When the Crab Pulsar was detected at the center of the Crab Nebula supernova remnant, astronomers knew pulsars had to be related to supernovae and the stellar core crushed to neutron degeneracy.




1 arcmin
Crab pulsar
a
10 arcsec
Pulsar in
"on" state
Pulsar in
"off" state
b

The Crab pulsar also pulses in visual light.

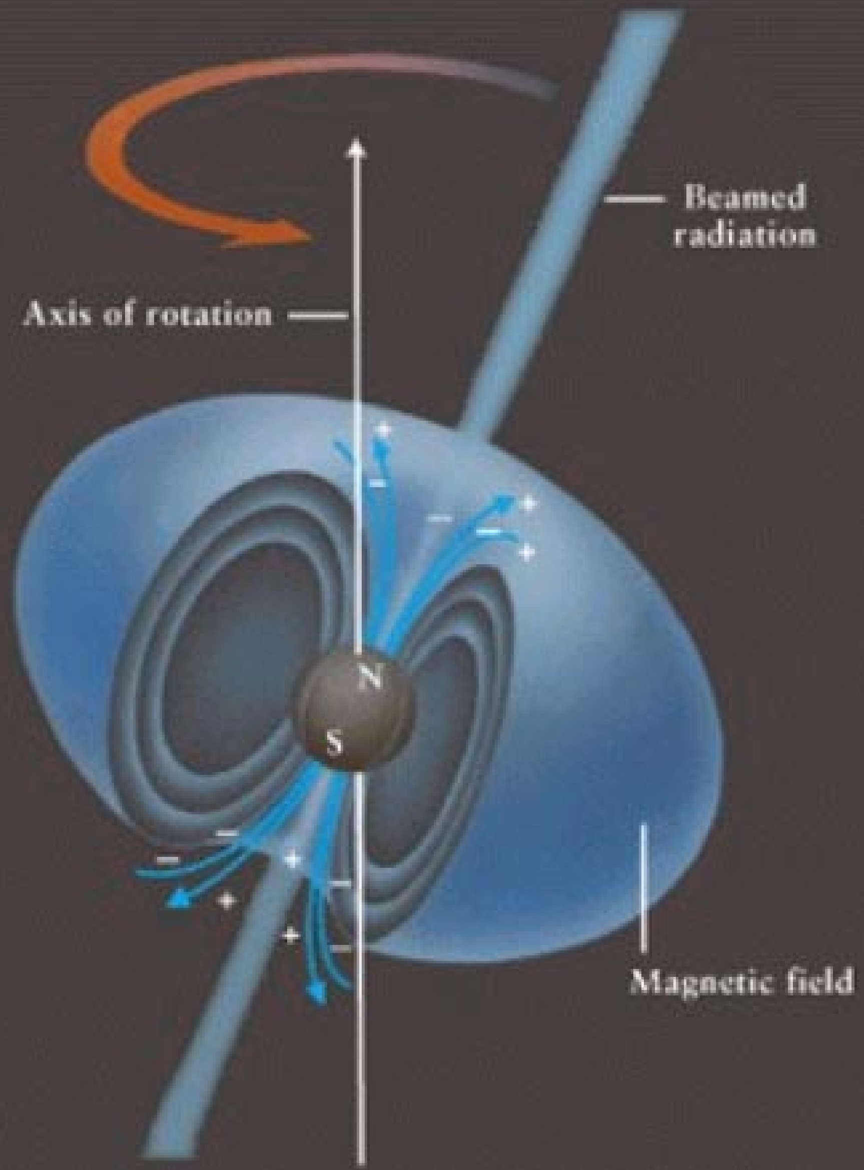


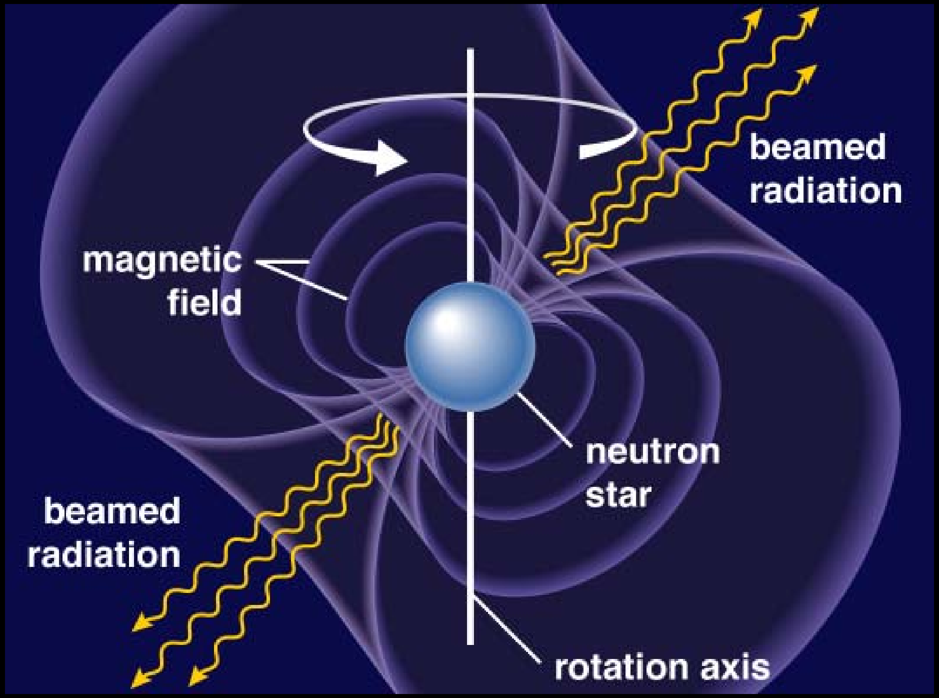


Pulsars are rapidly rotating neutron stars with intense magnetic fields.

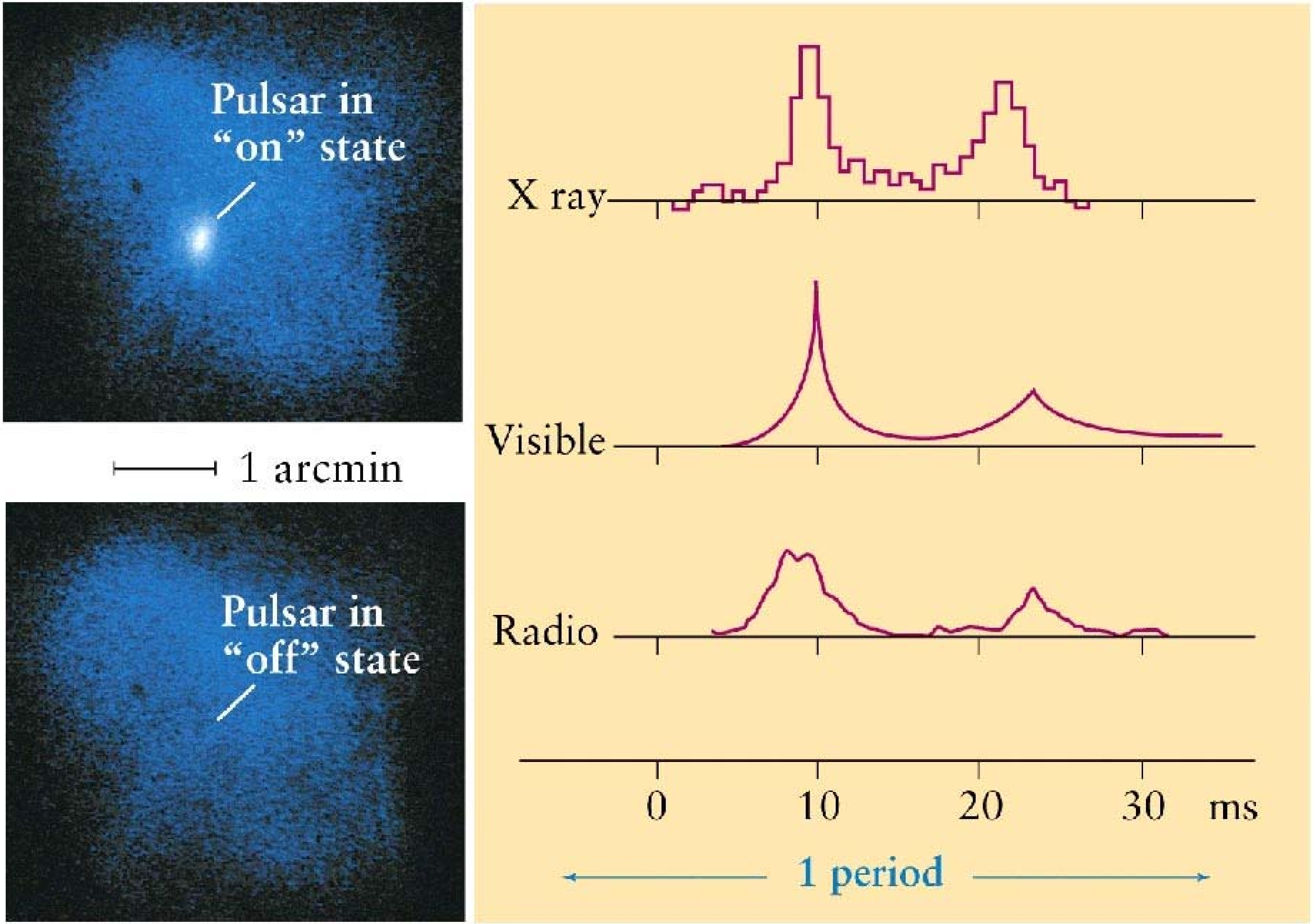
THE LIGHT HOUSE MODEL

A rotating magnetic field explains the pulses from a neutron star.





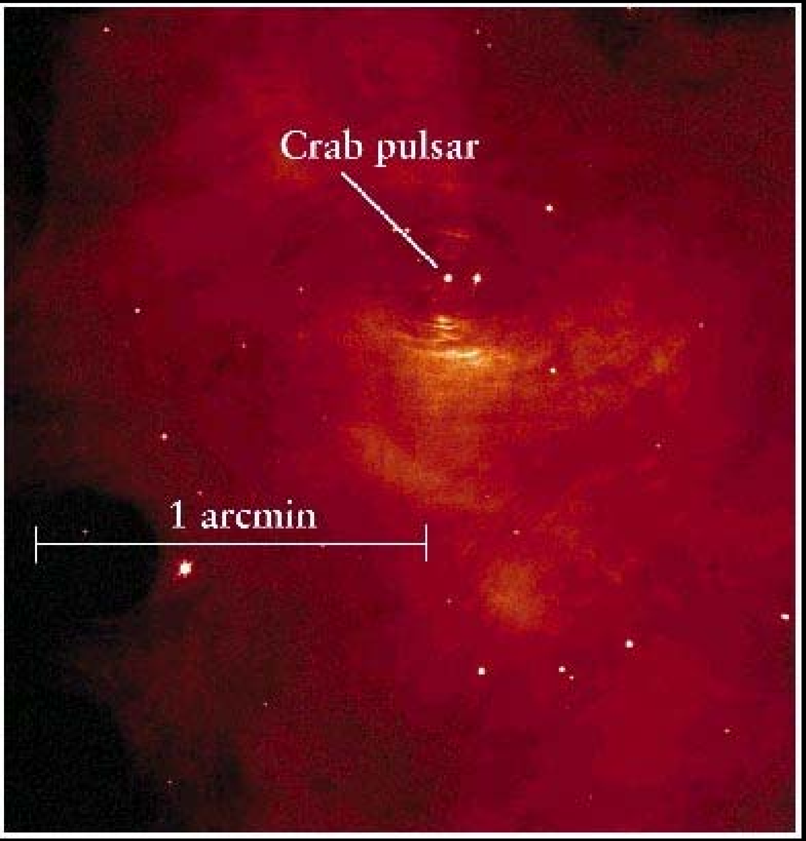
Pulsars are the lighthouses of Galaxy!



Pulsar in
"on" state
1 arcmin
Pulsar in
"off" state
X ray
Visible
Radio
0
10
20
30
ms
1 period

a

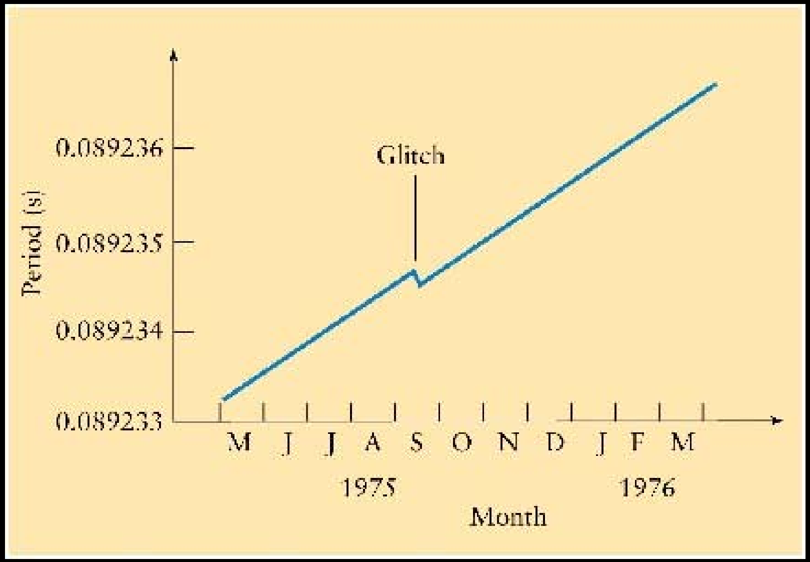
Pulsars gradually slow down as they radiate energy into space.



- The Crab Pulsar is slowing 3 x 10^{-8} seconds per day.
- Electrons moving in a circular path at enormously high speed release energy in the form of synchrotron radiation.
- The age of a pulsar can be measured by how slow it is currently spinning.

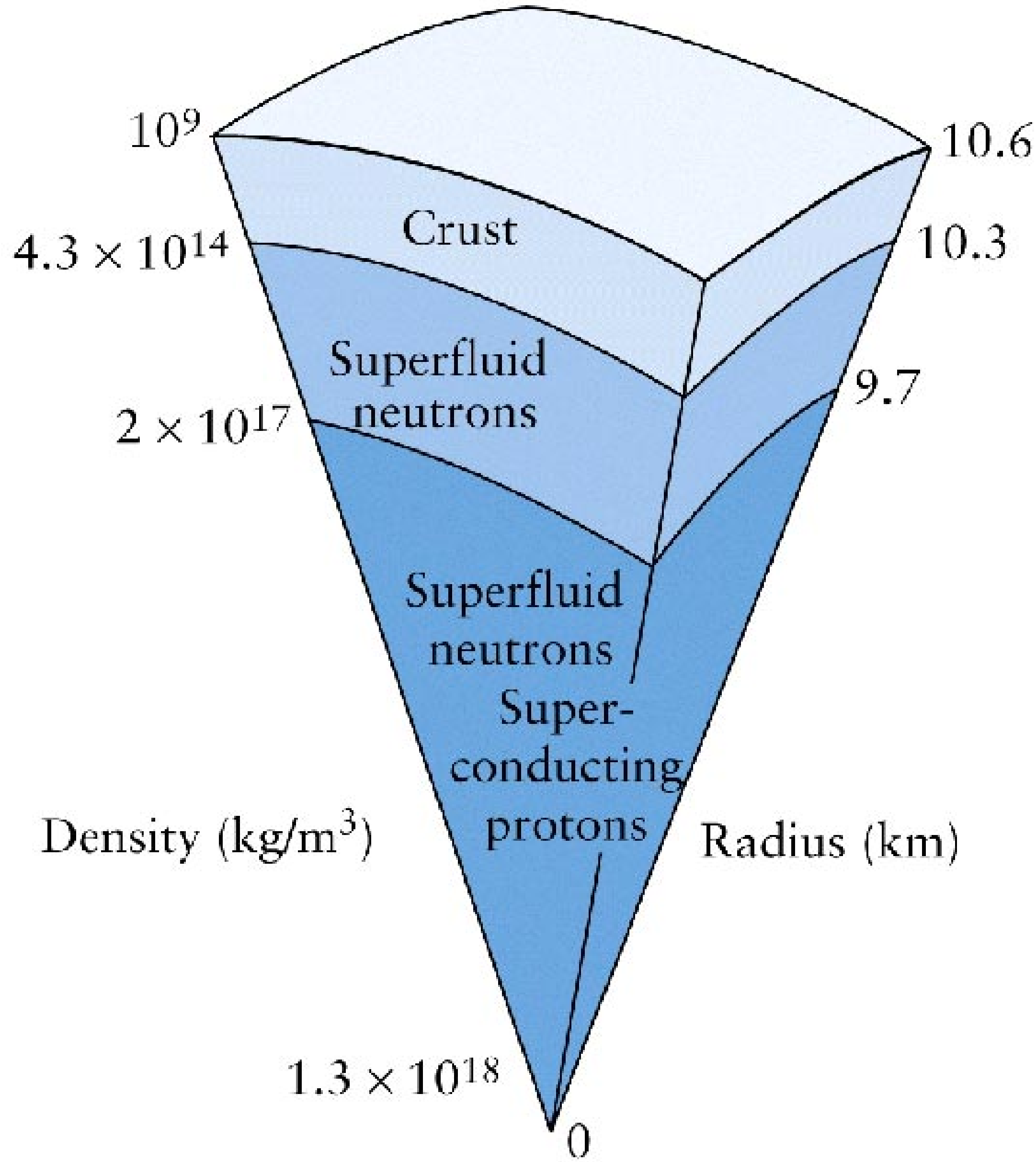
Superfluidity and superconductivity are among the strange properties of neutron stars.

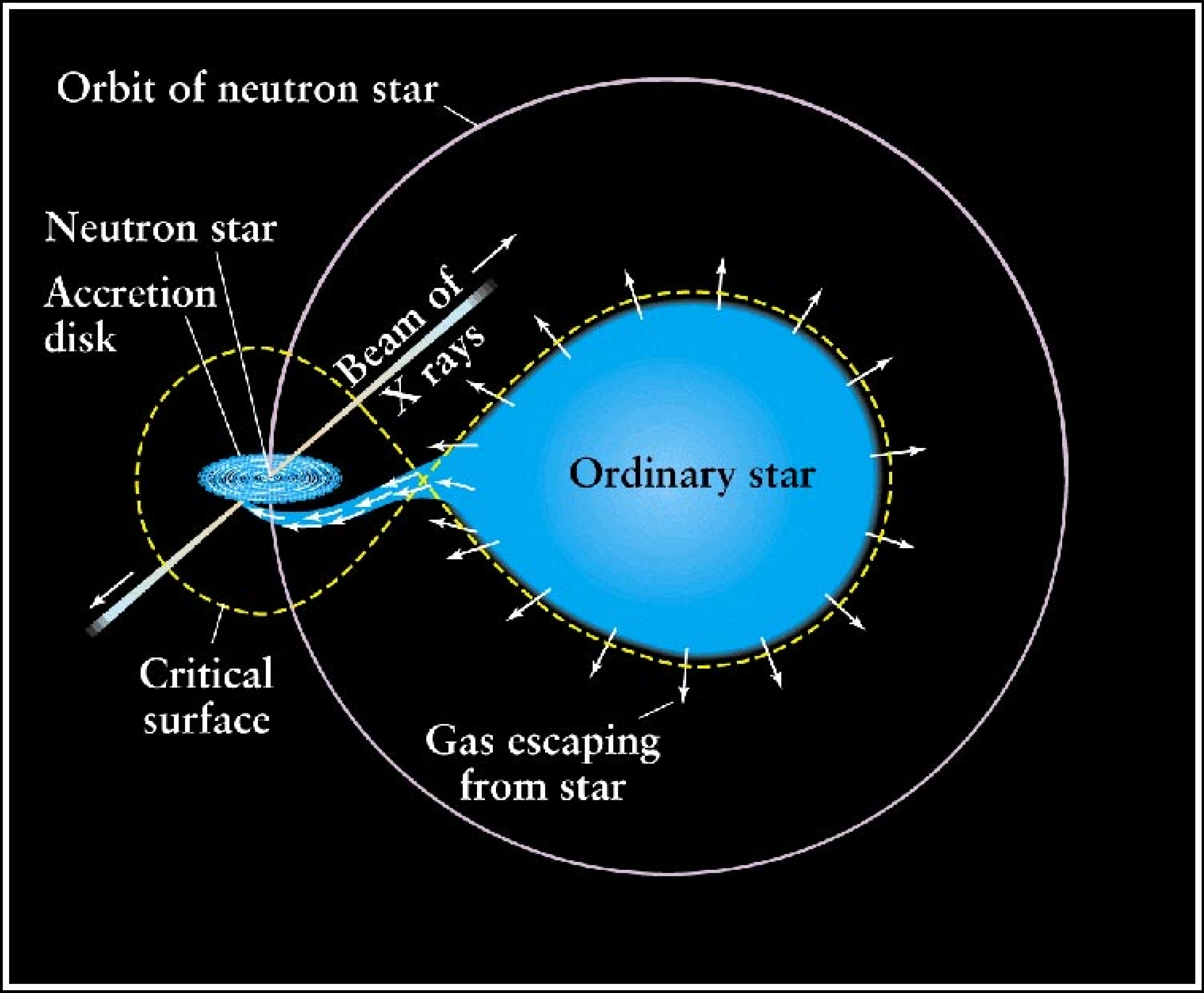
- Current models of neutron stars suggest that neutrons stars have a solid crust overlying a sea of neutrons that can flow without any friction whatsoever, called *superfluidity*.



- In addition to a general slowing of pulsars over time, they sometimes exhibit a sudden speed-up – called a *glitch* – probably caused by an instability in a slowing crust but a still rapidly rotating interior.

For a neutron star to have a magnetic field, it must also have a few protons scattered throughout to create the magnetic field.

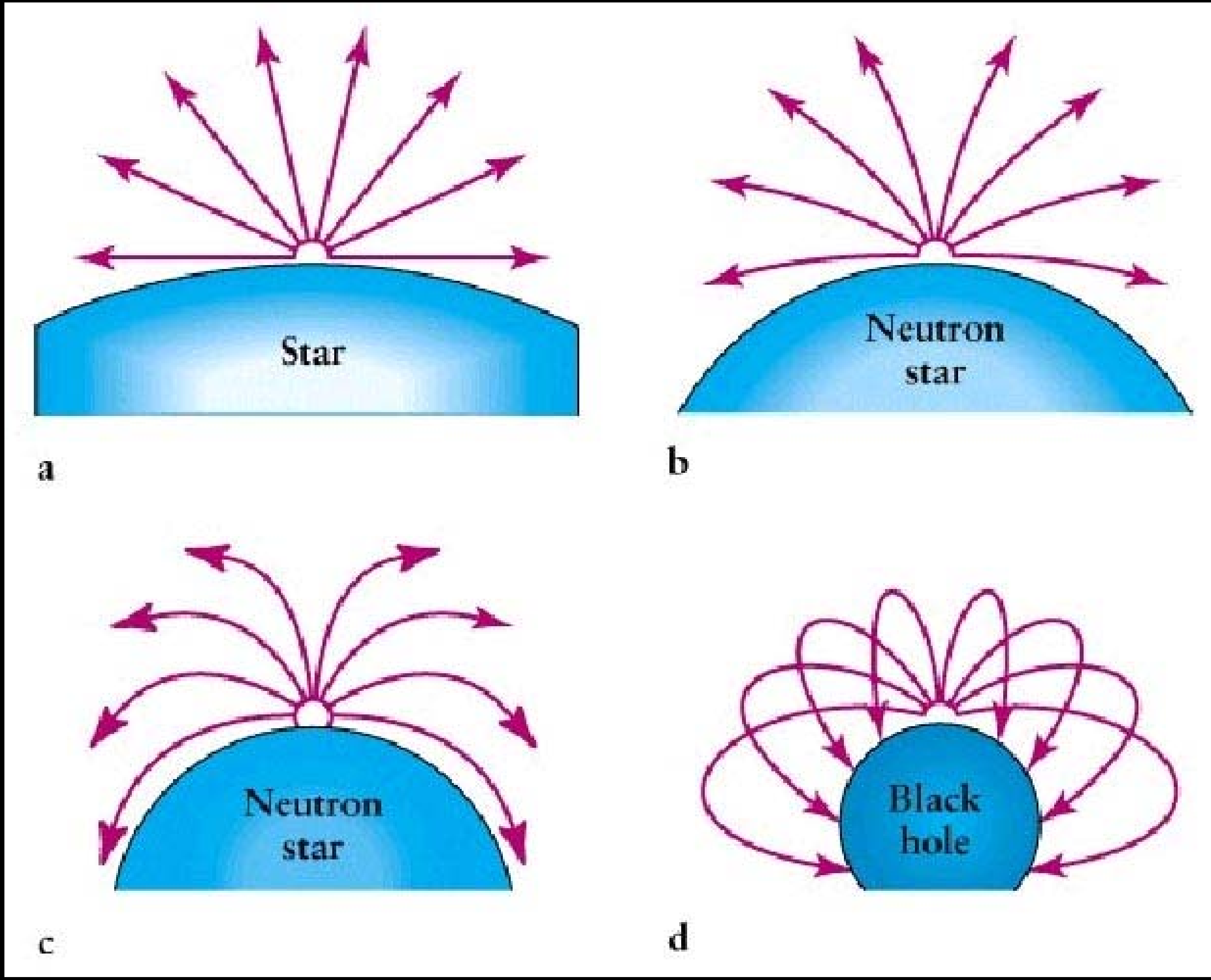



Orbit of neutron star
Neutron star
Accretion disk
Beam of X rays
Ordinary star
Critical surface
Gas escaping from star

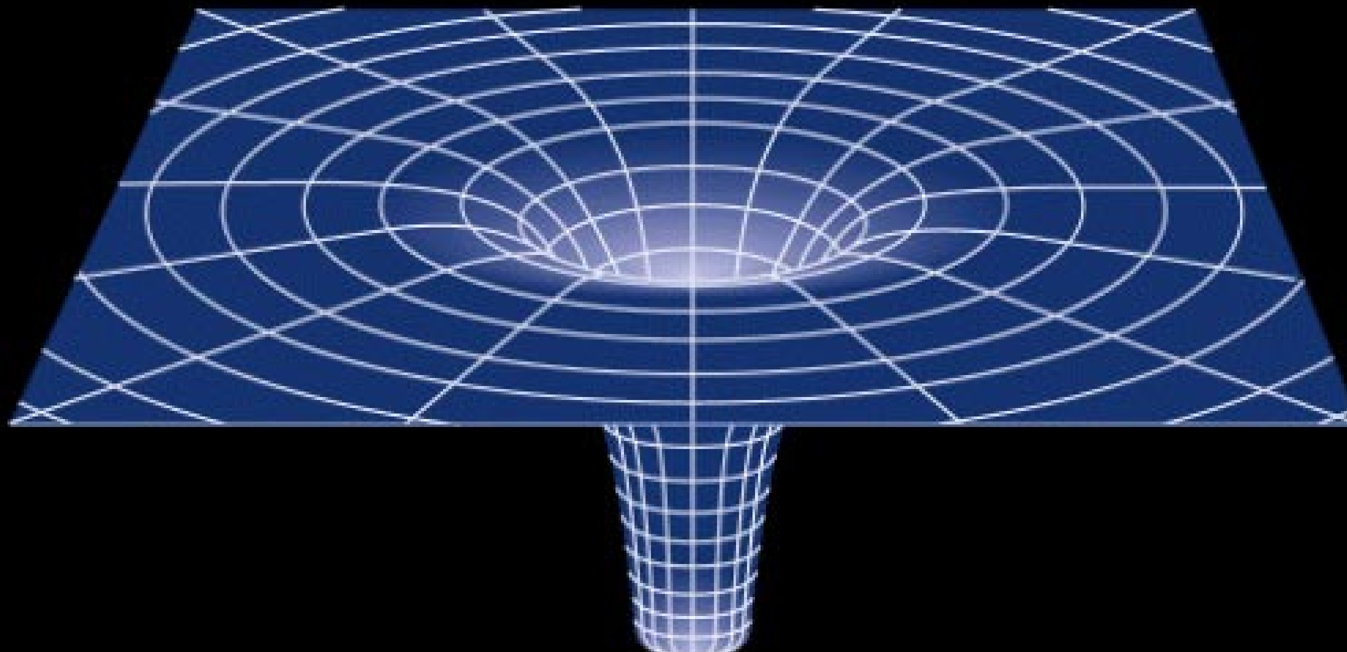
Like a white dwarf, a neutron star has an upper limit on its mass.

- White dwarfs will collapse if they exceed the Chandrasekhar limit of 1.4 $M_{\odot}$
- Neutron star upper mass limits are due to:
  - Degenerate nature of neutrons.
  - Strong nuclear force holding neutrons together.
- If a neutron star exceeds 3 $M_{\odot}$ then even photons cannot escape the star’s gravity and the object becomes a black hole .

If we apply *General Relativity* to a collapsing stellar core, we find that it can be sufficiently dense to trap light in its gravity.

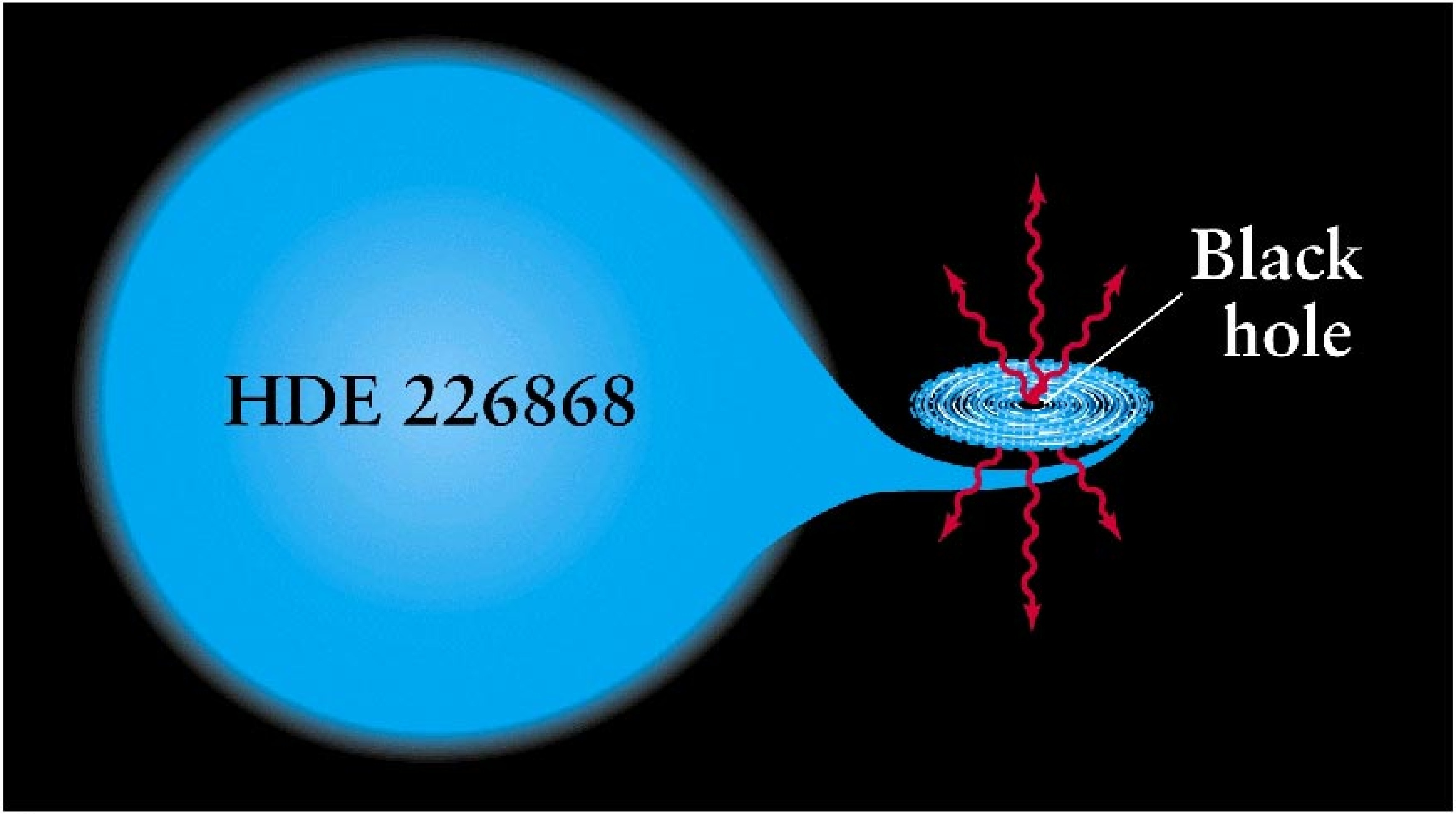


Spacetime should be distorted into an infinite well by a dense black hole



Certain binary star systems probably contain black holes

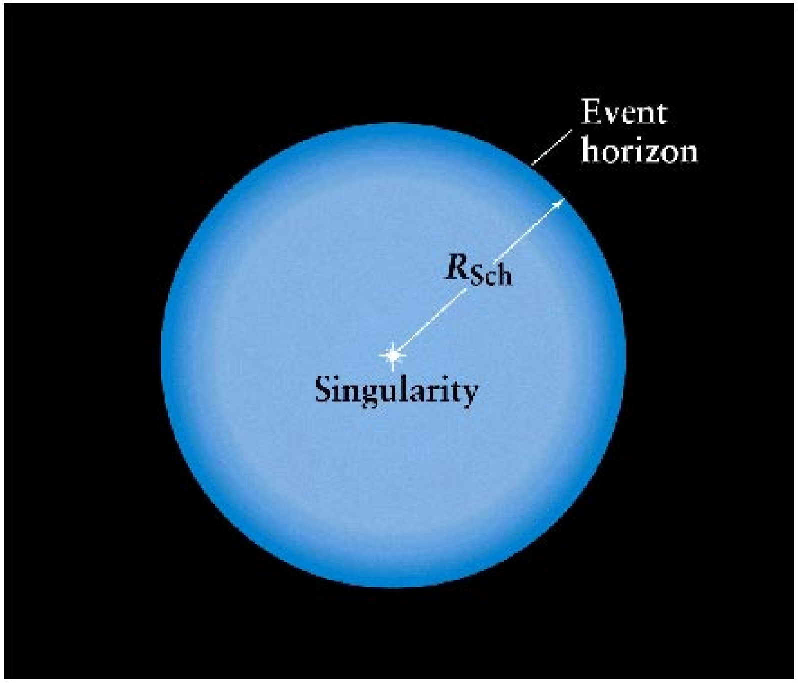
- Black holes cannot be seen because they do not emit nor reflect light
- Black holes that are in binary systems might be possible to detect
- As material races toward a black hole, it heats and emits X-rays



a

A nonrotating black hole has only a "center" and a "surface"

- The black hole is surrounded by an *event horizon* which is the sphere inside which light cannot escape
- The distance between the black hole and its event horizon is the *Schwarzschild radius* ($R_{Sch} = 2GM/c^2$)
- The center of the black hole is a point of infinite density and zero volume, called a *singularity*

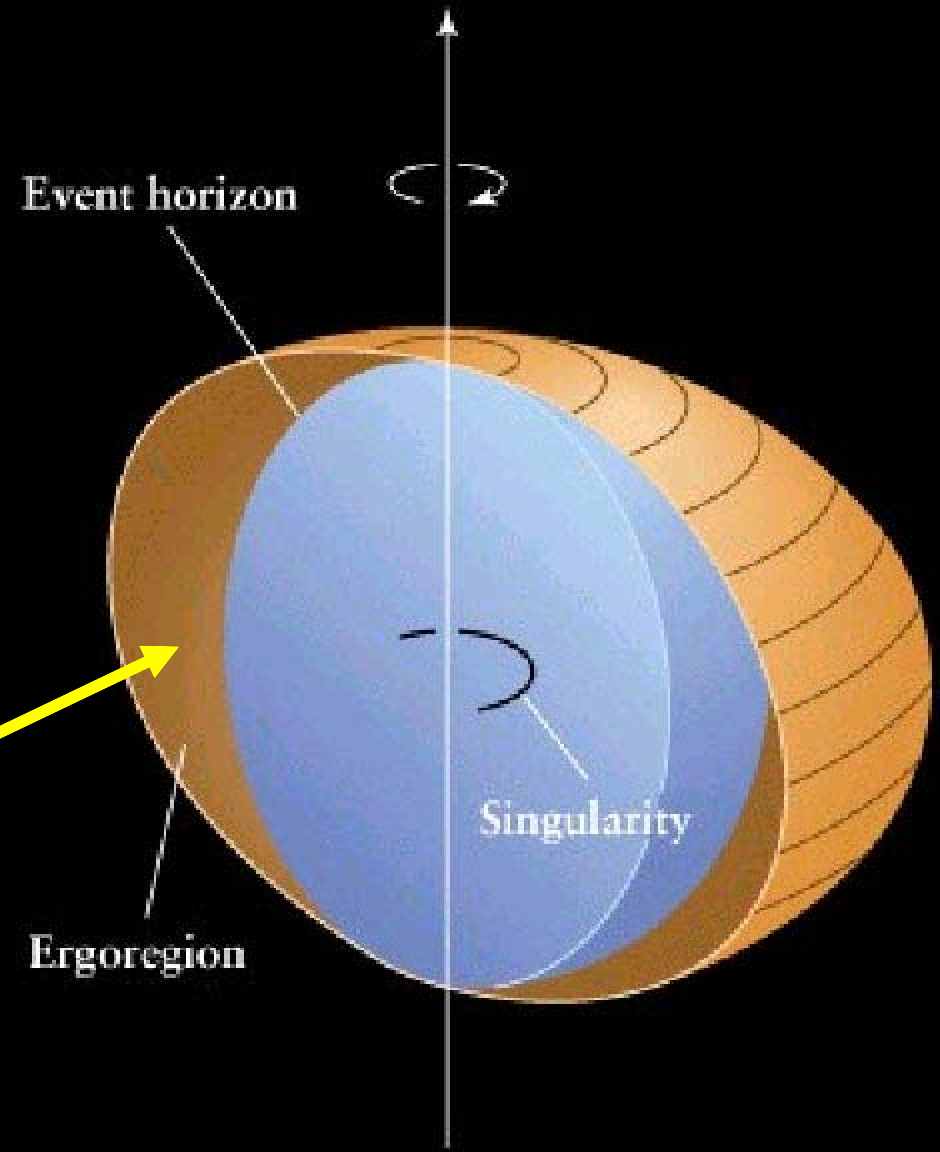


Just three numbers completely describe the structure of a black hole

Most properties of matter vanish when matter enters a black hole, such as chemical composition, texture, color, shape, size, distinctions between protons and electrons, etc

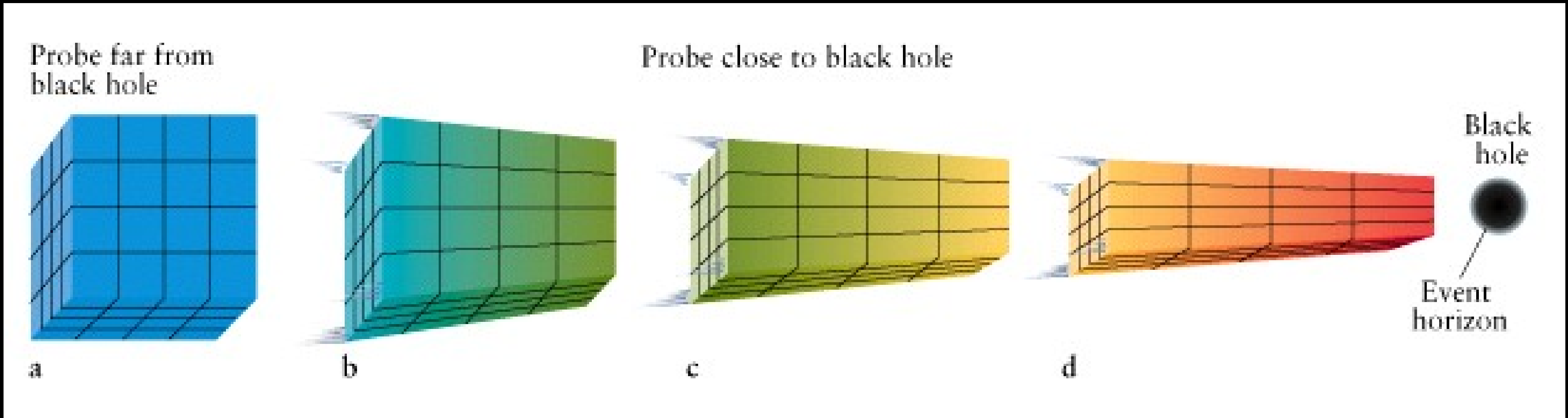
- Mass
  - As measured by the black hole's effect on orbiting bodies, such as another star
- Total electric charge
  - As measured by the strength of the electric force; probably quite small (it can have no magnetic field)
- Angular momentum
  - If the stellar core that collapsed was spinning, we expect the black hole to be spinning quite quickly

Structure of a Kerr (*Rotating*) Black Hole



In the ***Ergoregion***, nothing can remain at rest as spacetime here is being pulled around the black hole.

Seen from outside, falling into a black hole is an infinite voyage as gravitational tidal forces pull spacetime in such a way that time becomes infinitely long



Black holes may evaporate

Virtual particles that appear in pairs near an event horizon may not be able to mutually annihilate each other if only one manages to survive a trip along the event horizon.