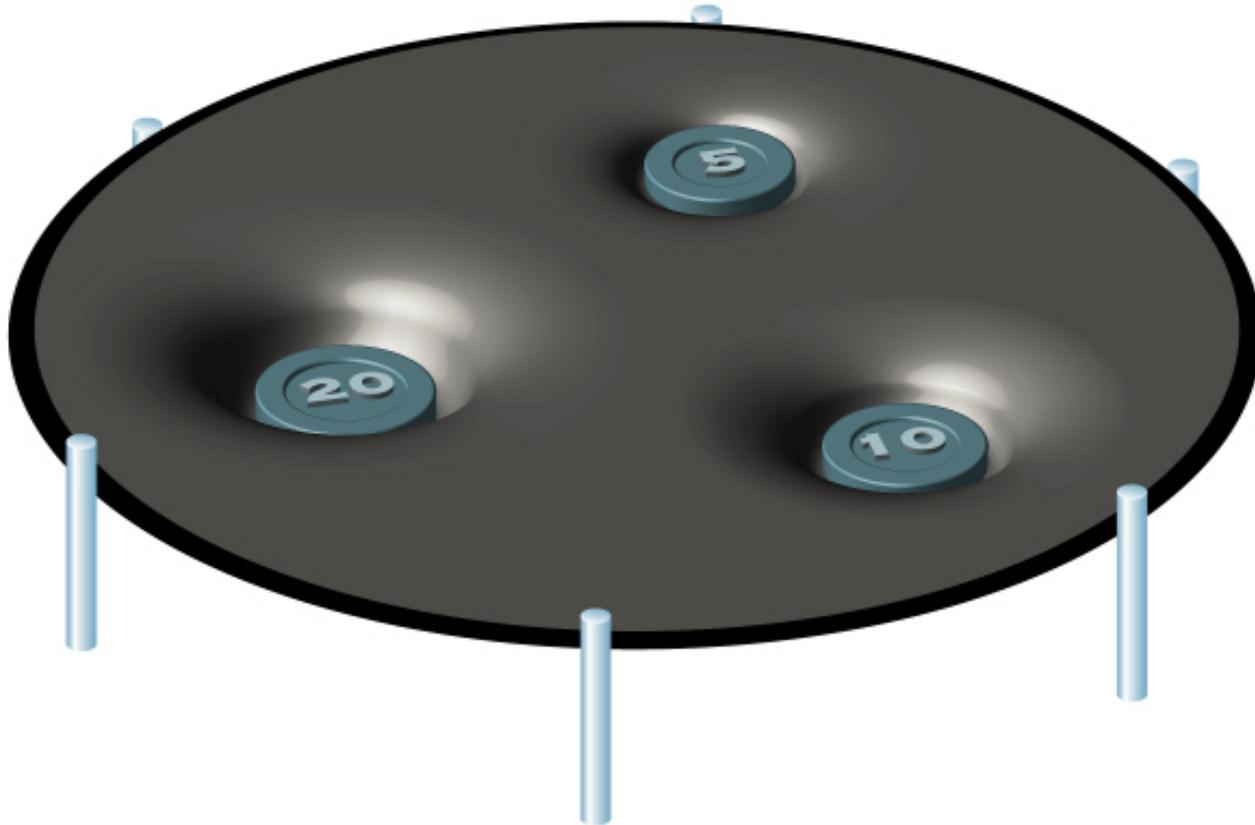


Lecture 10 Cosmology

Expansion of the Universe & Hot Big Bang



Momentum-Energy distorts Spacetime!

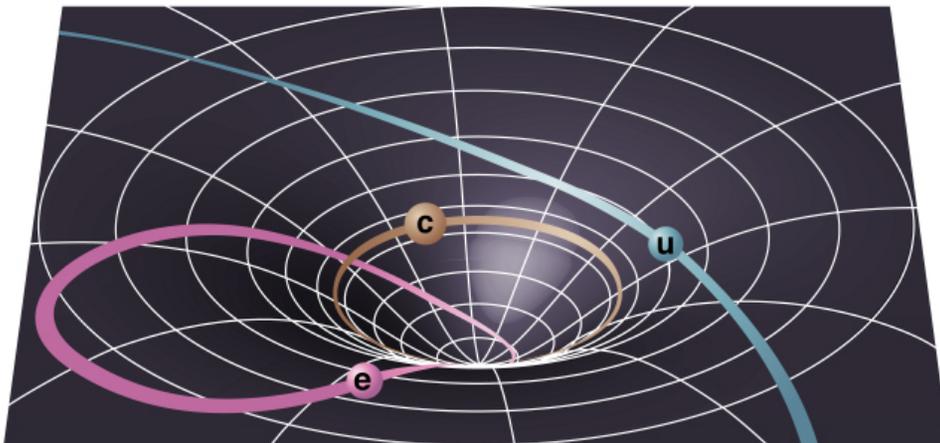


- Matter distorts spacetime like weights on a taut rubber sheet.
- The greater the mass, the greater the distortion of spacetime.

Momentum-Energy and Spacetime

- According to Newton, all bodies with mass exert a gravitational force on each other.
 - even Newton had problems accepting this concept of “action at a distance”
- General relativity removes this concept.
 - Momentum-Energy causes spacetime to curve
 - the greater the mass, the greater the distortion of spacetime
 - curvature of spacetime determines the paths of freely moving objects

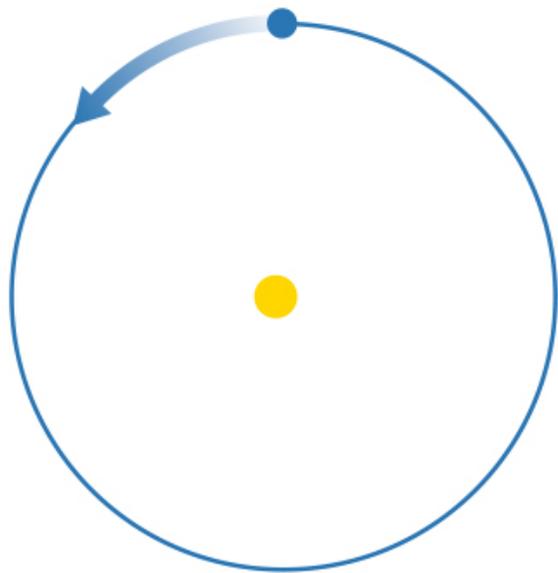
c circular orbit
e elliptical orbit
u unbound orbit



- Orbits can now be explained in a new way.
 - an object will travel on as straight a path as possible through spacetime

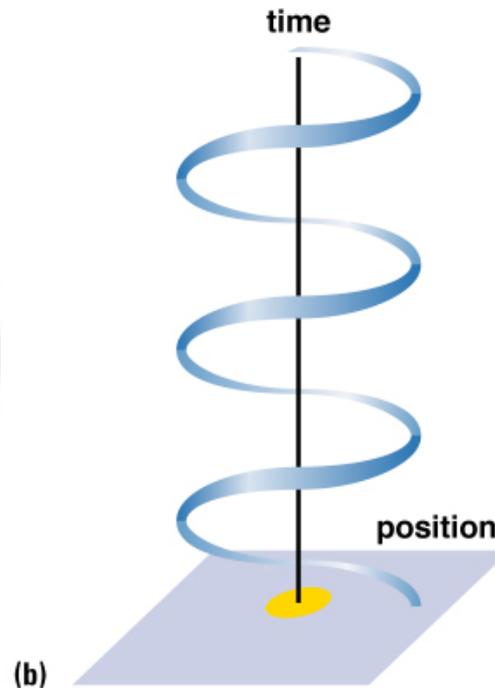
Orbits in Spacetime

- The “rubber mat” analogy shows only an object’s position in two dimensions of space.
 - Earth returns to the same position in space (*w.r.t.* the Sun) each year
 - Earth does not return to the same position in spacetime each year
 - Earth must also move forward in time



(a)

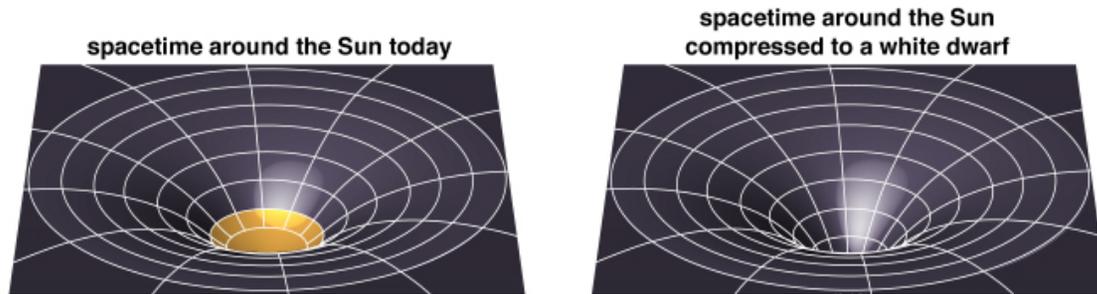
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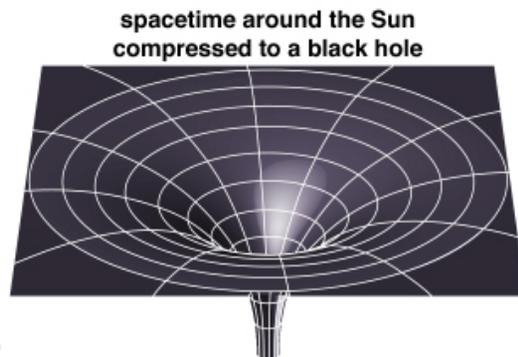
(b)

The Strength of Gravity

- The more that spacetime curves, the stronger (Newtonian) gravity becomes.
- Two basic ways to increase gravity/curvature of spacetime:
 - increased mass results in greater curvature at distances away from it
 - curvature is greater near the object's surface for denser objects
 - for objects of a given mass, this implies smaller objects

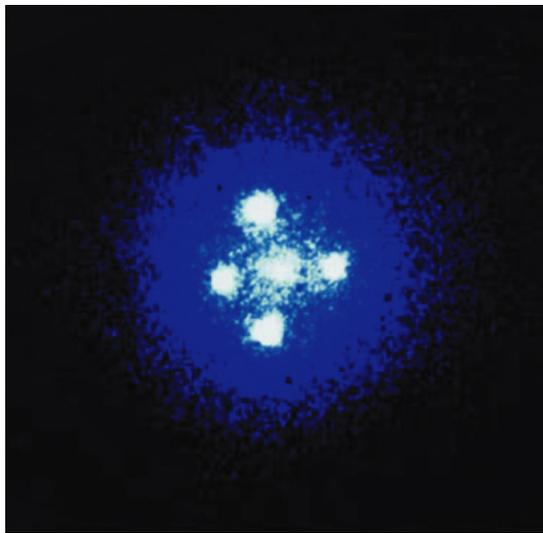
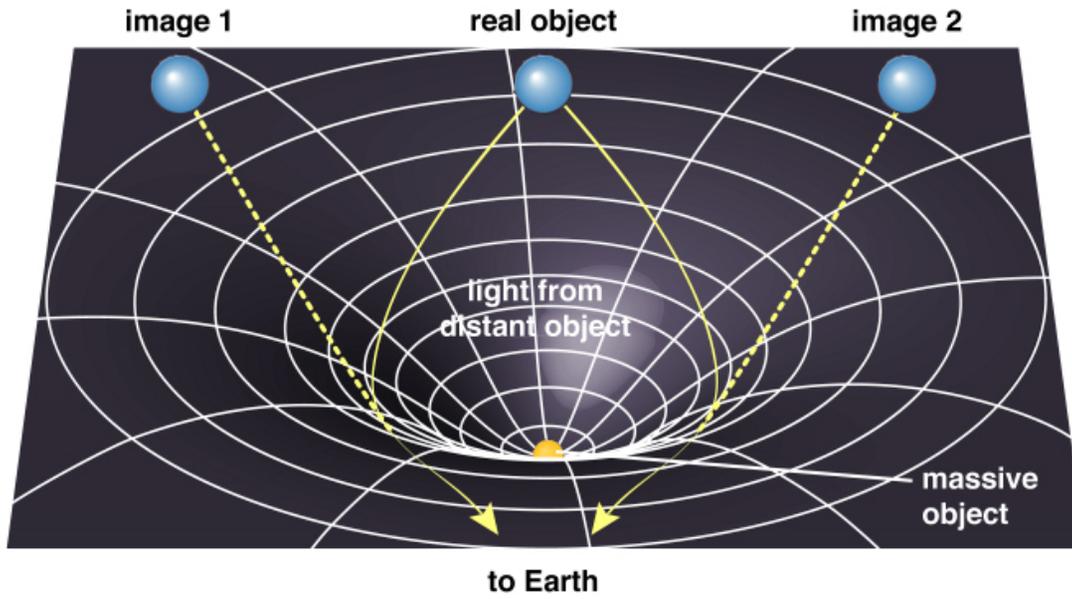


(a)

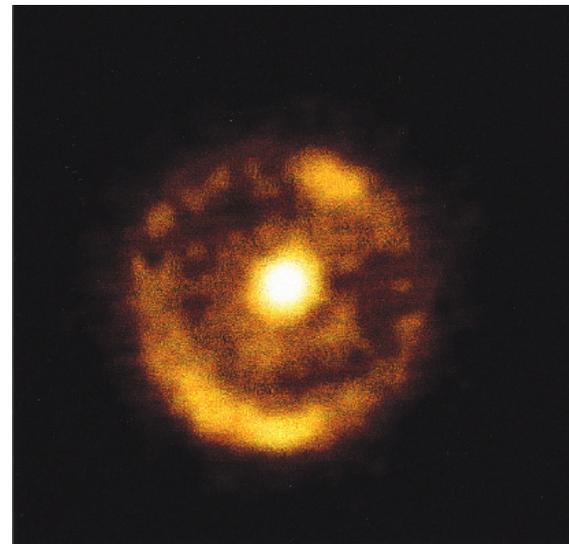


(b)

- All three objects impose the same curvature at a distance.
- White dwarf imposes steeper curvature at Sun's former position.
- Black hole punches a hole in the fabric of spacetime.
- Nothing can escape from within the event horizon.



Einstein Cross



Einstein ring

Weak equivalence principle

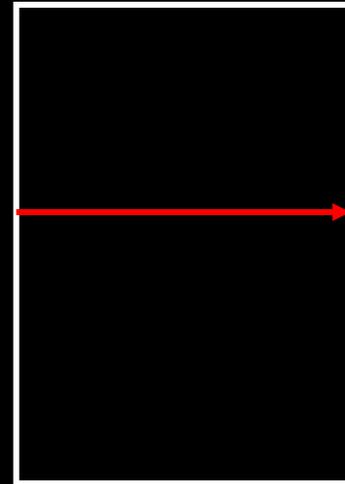
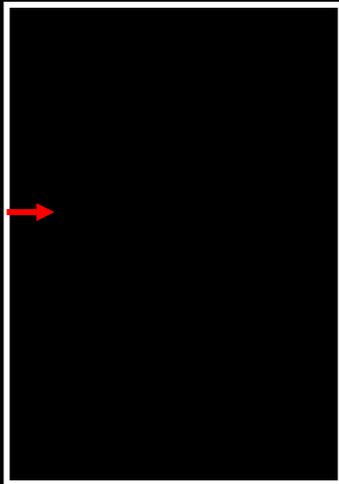
The **laws of mechanics** are precisely the same in all inertial and freely falling frames. In particular, gravity is completely indistinguishable from any other acceleration.

Strong equivalence principle

The **laws of physics** are precisely the same in all inertial and freely falling frames, there is no experiment that can distinguish them.

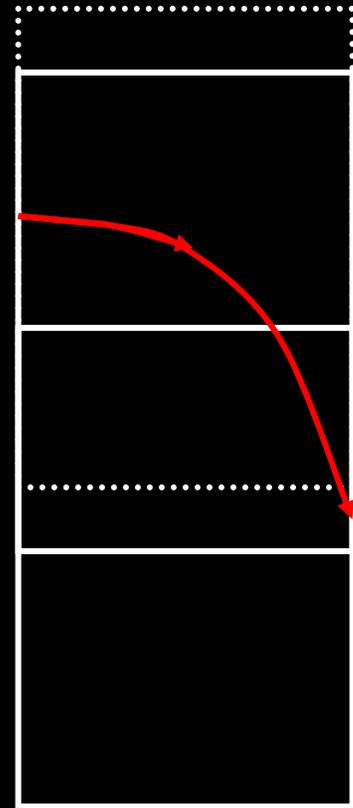
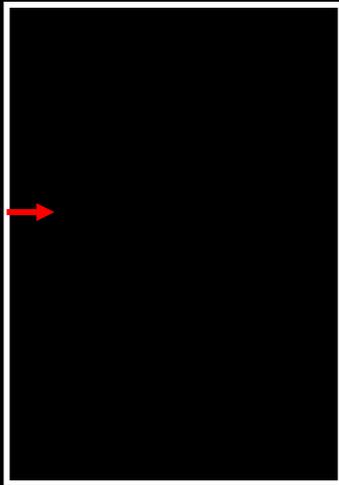


Consequences of the equivalence principle: mass bends light



Observer in freely falling reference frame

Consequences of the equivalence principle: mass bends light



Outside Observer

So does the existence of tidal forces violate the equivalence principle ?

- there is no freely falling frame of reference in which gravity vanishes **globally**
- there is a freely falling frame of reference in which gravity vanishes **locally**
- equivalence principle holds for **small labs**, “small” in comparison to distances over which the gravitational field changes significantly.
- spacetime is locally flat

Towards a new theory for gravity ...

Requirements:

- it should locally fulfill the equivalence principle
- it should relate geometry of space to the distribution of mass and energy
- it should be locally flat
- it should reduce to Newtonian gravity for small velocities (compared to c) and for weak gravitational fields

The entire Universe in one line

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Geometry of
spacetime
(Einstein tensor)

Distribution of
mass and energy
in the universe
(stress-energy tensor)

Why is general relativity (GR) difficult ?

- conceptually difficult (relativity of space and time, curvature of spacetime)
- set of 10 coupled partial differential equations
- non-linear (solutions do not superpose)
- space and time are part of the solution

⇒ exact solution known only for a very few simple cases

The darkness of the night sky tells us that the universe expands (solution to *Olber's paradox*)

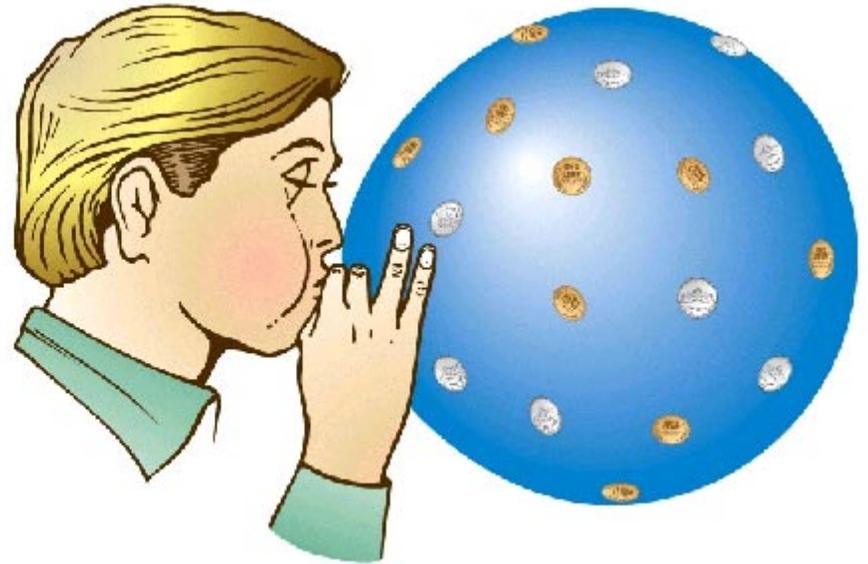


- If space goes on forever, then eventually every space should be filled with light, yet that's not what we see when we look up.
- Olbers's paradox is that the sky is actually dark in places.
- However, we live in an expanding universe and all of the light in the universe has not yet reached us.

The universe is expanding

- Hubble's law shows that the more distant galaxies have higher recessional velocities.
- Hubble's law is the same in all directions (*isotropic*).
- Hubble's law allows to “play the scenario backwards” and determine an age of the universe.
- Doppler redshifts are caused by an object's motion whereas *cosmological redshifts* are caused by the expansion of spacetime itself.

The universe is expanding



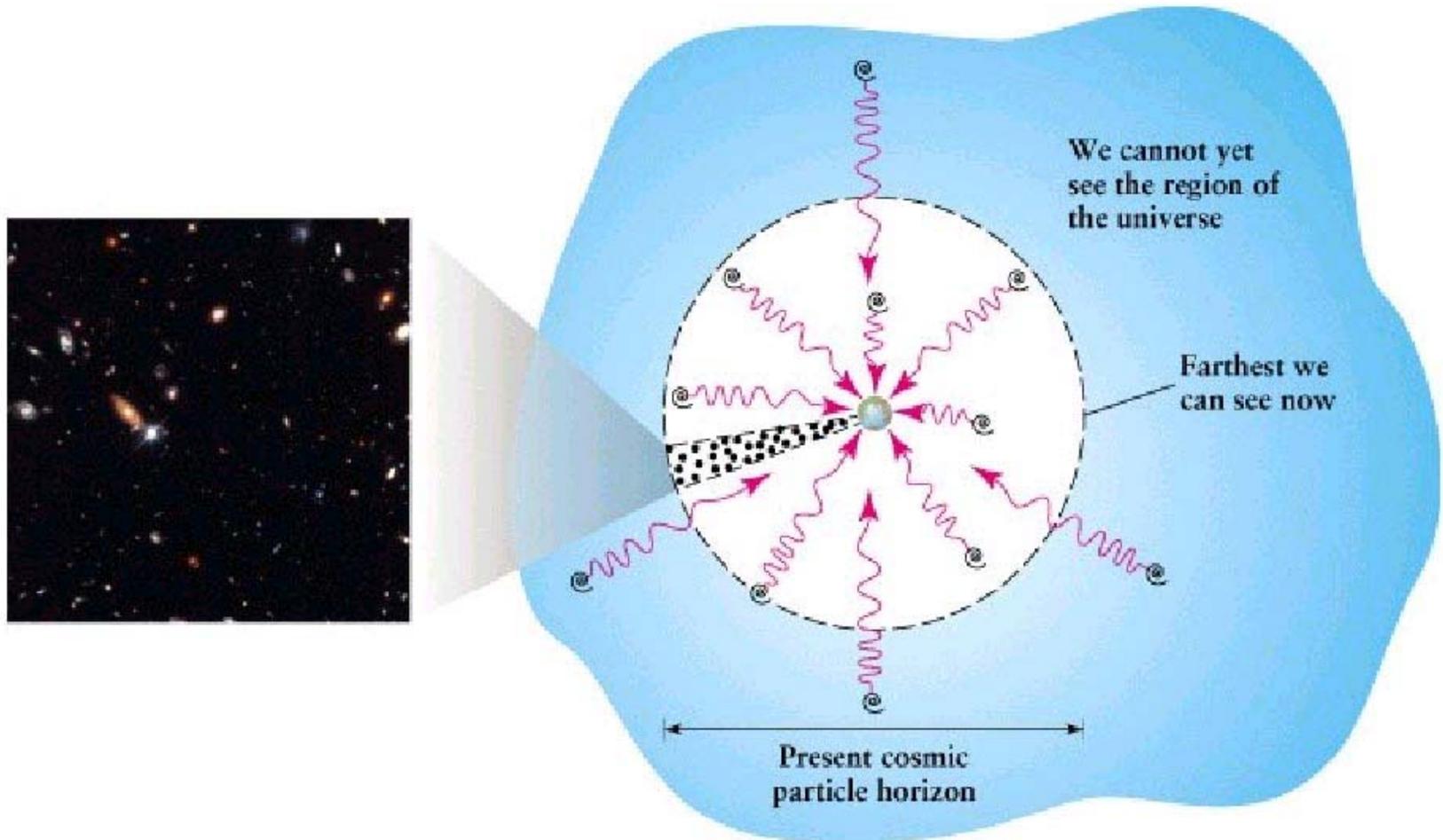
The expanding universe emerged from a cataclysmic event called the Big Bang.

- In the 1940s, based on Hubble's Law, George Gamow proposed the universe began in a colossal explosion.
- In the 1950s, the term BIG BANG was coined by an unconvinced Fred Hoyle.
- In the 1990s, there was an international competition to rename the BIG BANG with a more appropriate name, but no new name was selected.

BIG BANG is a relatively simple idea

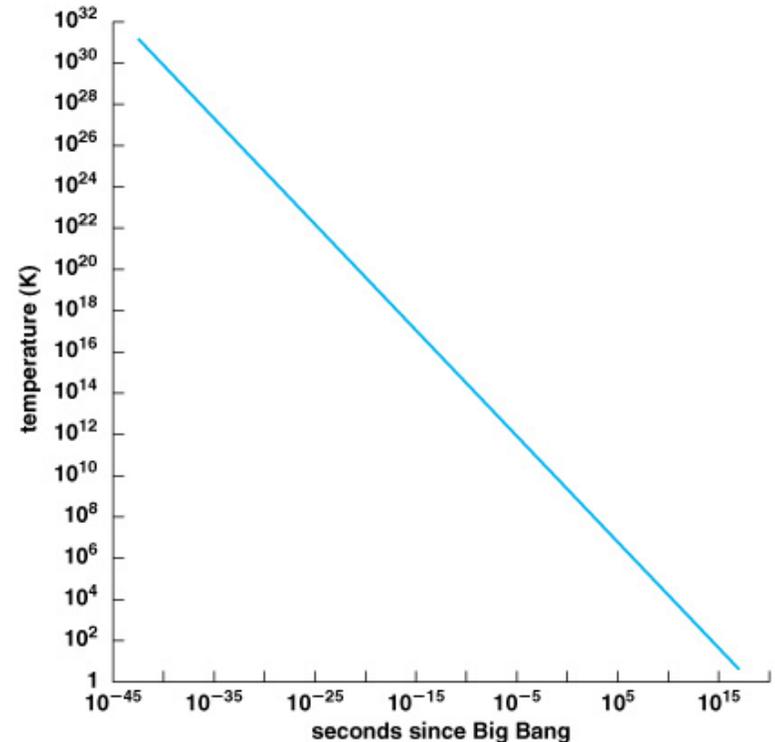
- If the universe is expanding, it must have been smaller in the past.
- If it was smaller in the past, then something must have made it begin to expand.
- This “event” is called the BIG BANG.
- The age of the universe is simply the separation distance of the most distant galaxies divided by their recessional velocities.
- Current figures place the age about 13.7 billion years.

The farther we look into space, the farther back in time we are seeing.



Conditions in the Early Universe

- The most distant galaxies we observe come from a time when the Universe was a few billion years old.
- The cosmic microwave background prevents us viewing light from before the Universe was 380,000 years old.
- So how do we know what conditions were like at the beginning of time?
- We know the the conditions & expansion rate of the Universe today.
- By running the expansion backwards:
 - we can predict the temperature & density of the Universe anytime in its history using basic physics
 - we study how matter behaves at high temperatures & densities in laboratory experiments
 - current experimental evidence provides info on conditions as early as 10^{-10} sec after the Big Bang



The universe was a hot, opaque plasma during its first 400,000 years.

- Everything in the early universe falls into two categories: *matter* or *energy*.

- Mass density of radiation, ρ_{rad}

$$\rho_{\text{rad}} = 4\sigma T^4 / c^3 \quad \sigma = 5.67 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$$

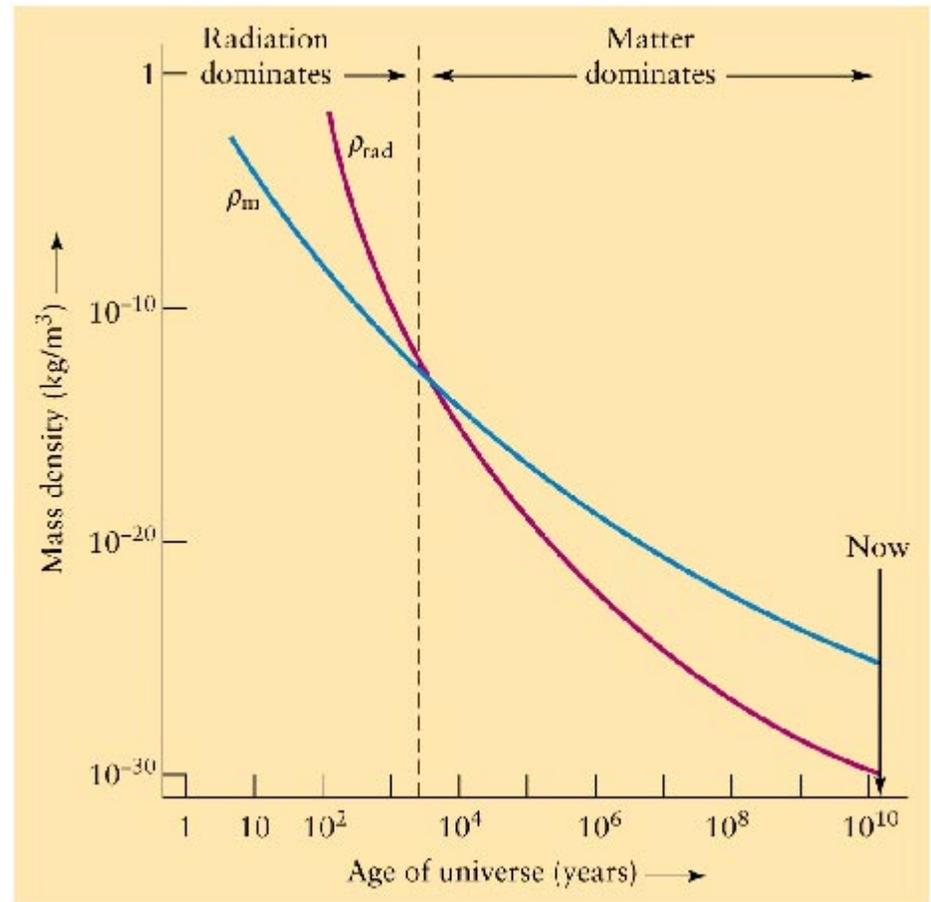
$$[\text{for } T = 2.725\text{K, } \rho_{\text{rad}} \text{ is } 4.6 \times 10^{-31} \text{ kg/m}^3]$$

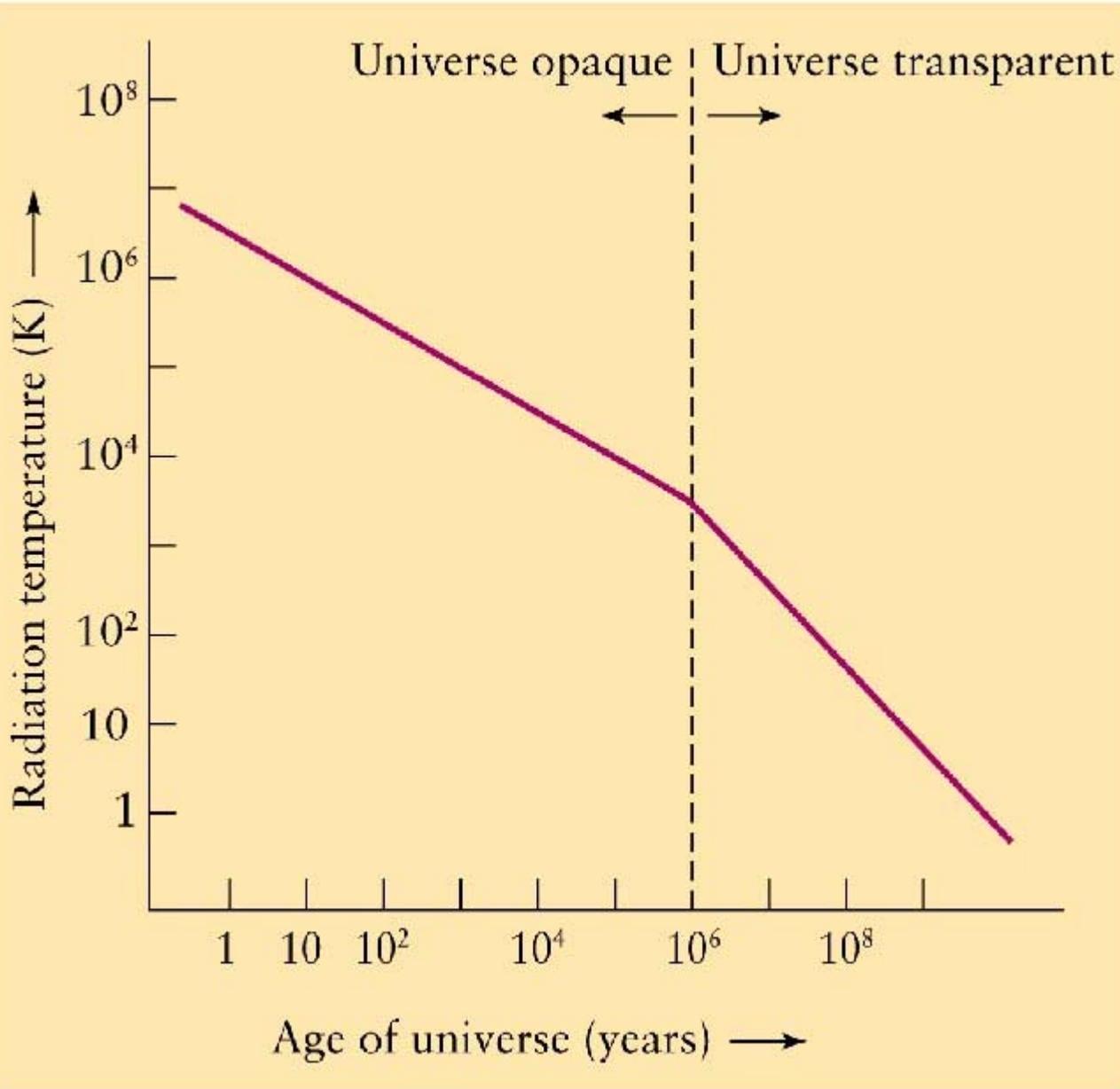
- Average density of matter, ρ_{mass}

Present Day ρ_{mass} is about 2 to 4 x 10⁻²⁷ kg/m³

The universe was a hot, opaque plasma during its first 400,000 years.

The early radiation dominated universe became today's matter dominated universe.





BANG!

Planck Era ($t < 10^{-43}$ sec)

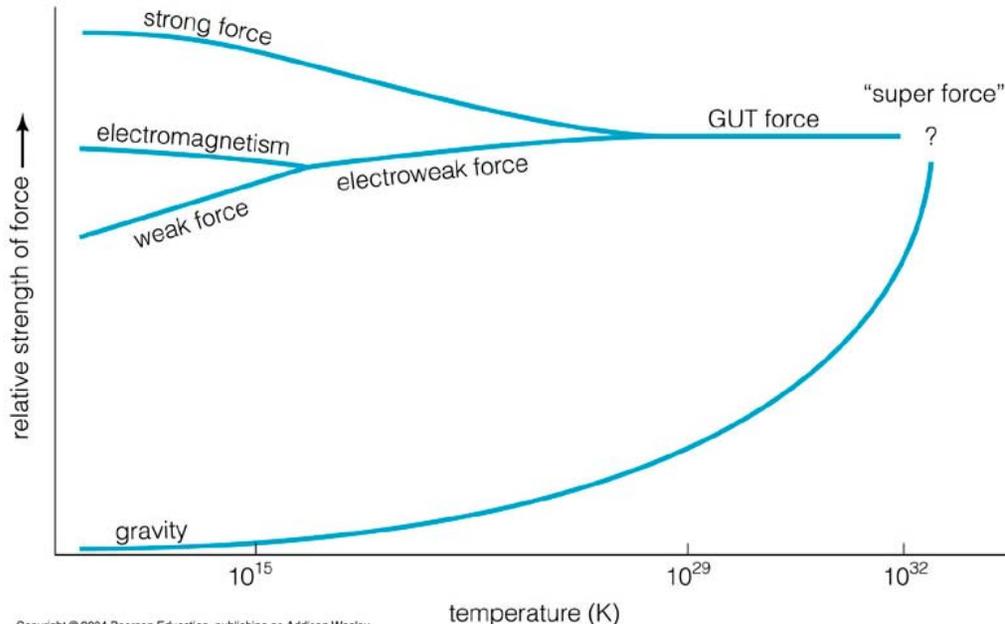
- This era, the “first instant”, lasted for 10^{-43} sec.
- Because we are as yet unable to link...
 - quantum mechanics (our successful theory of the very small)
 - general relativity (our successful theory of the very large)
- We are powerless to describe what happened in this era.
- 10^{-43} sec after the Big Bang is as far back as our current science will allow us to go.
- Perhaps all four natural forces were unified during this era.

GUT Era ($10^{-43} < t < 10^{-38}$ sec)

- The Universe may have contained only two natural forces:
 - gravity
 - Grand Unified Theory (GUT) force
 - = electromagnetic + strong (nuclear) + weak forces unified
- This lasted until the Universe was 10^{-38} sec old.
 - at this time, the Universe had cooled to 10^{29} K
 - the strong force “froze out” of the GUT force
 - the energy released by this caused a sudden and dramatic **inflation** of the size of the Universe

Electroweak Era ($10^{-38} < t < 10^{-10}$ sec)

- The Universe contained three natural forces:
 - gravity, strong, & electroweak
- This lasted until the Universe was 10^{-10} sec old.
 - at this time, the Universe had cooled to 10^{15} K
 - the electromagnetic & weak forces separated

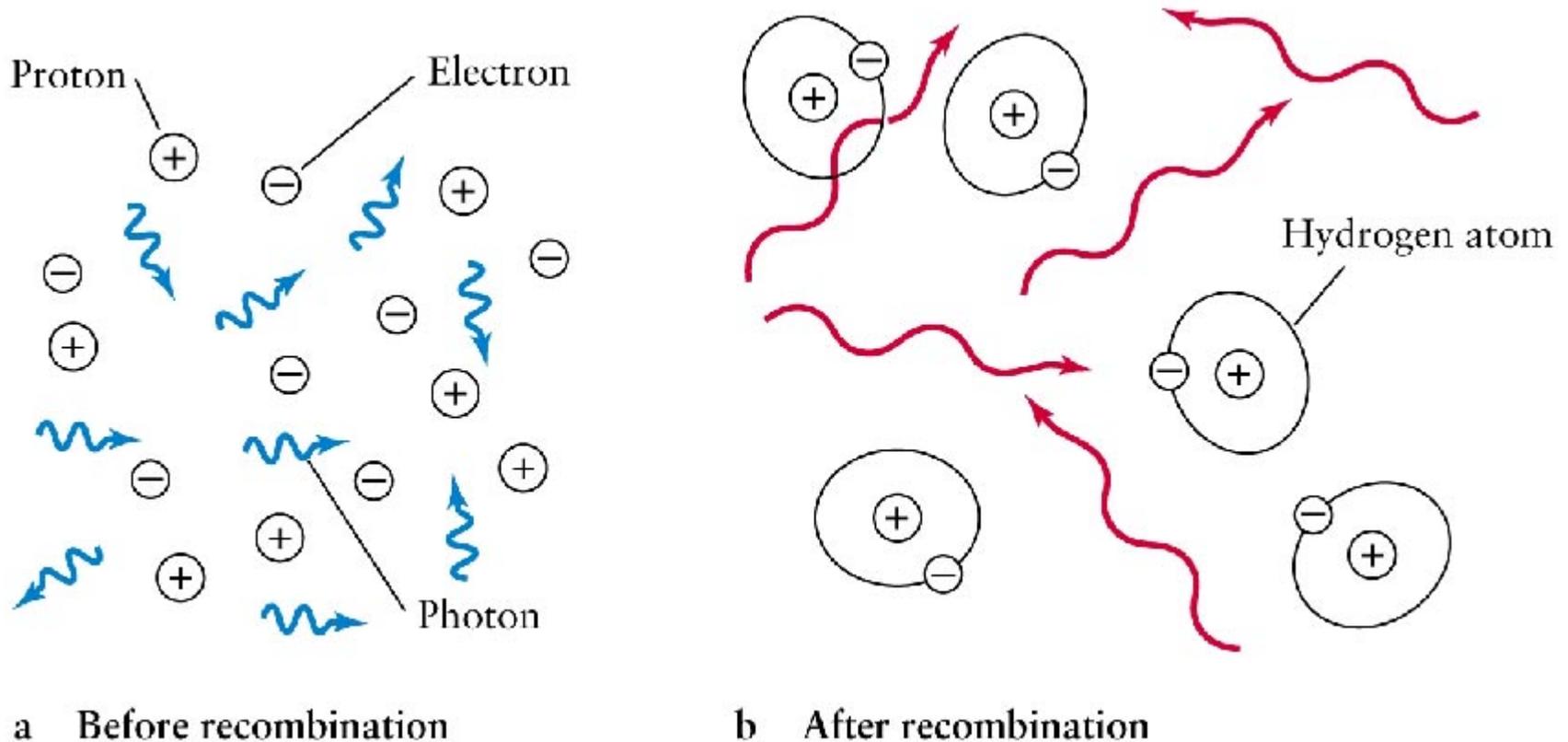


- This was experimentally verified in 1983:
 - discovery of W & Z bosons
 - electroweak particles predicted to exist above 10^{15} K

Particle Era ($10^{-10} < t < 10^{-3}$ sec)

- The four natural forces were now distinct.
- Particles were as numerous as photons.
- When the Universe was 10^{-4} sec old...
 - quarks combined to form protons, neutrons, & their anti-particles
- At 10^{-3} sec old, the Universe cooled to 10^{12} K.
 - protons, antiprotons, neutrons, & antineutrons could no longer be created from two photons (radiation)
 - the remaining particles & antiparticles annihilated each other into radiation
 - slight imbalance (why?) in number of protons & neutrons allowed matter to remain
- Electrons & positrons are still being created from photons.

The universe was a hot, opaque plasma during its first 400,000 years.

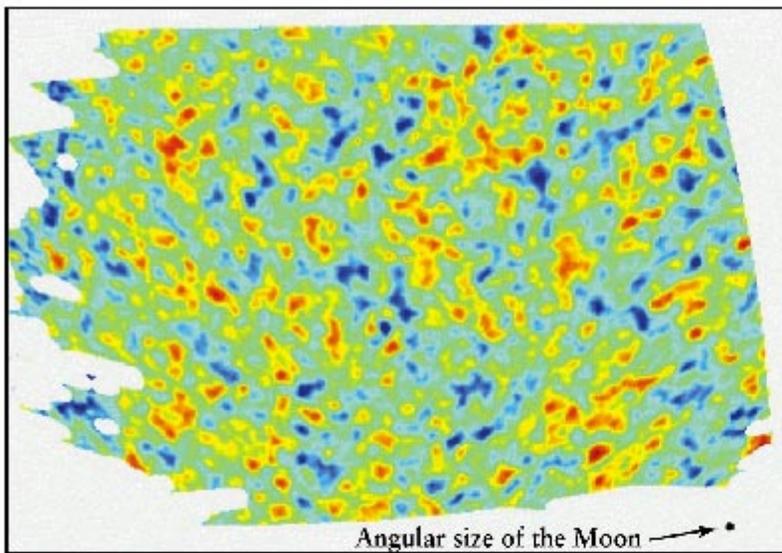


At $t = 400,000$ years, the universe was finally cool enough from its initial *primordial fireball* that electrons and protons could combine to form atoms (era of recombination).

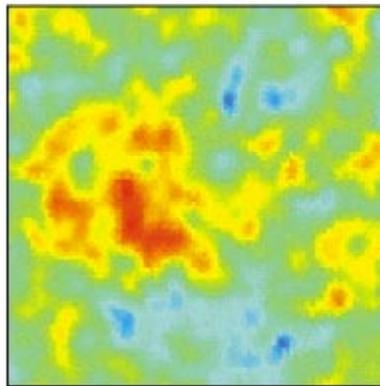
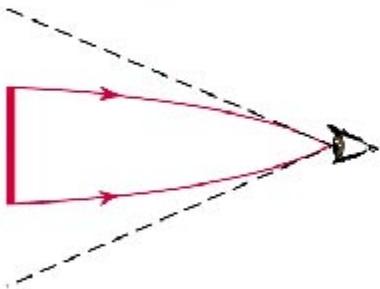
The shape of the universe indicates its matter and energy content.

The shape of our universe depends on the combined average mass density of all forms of matter and energy. The three possibilities are:

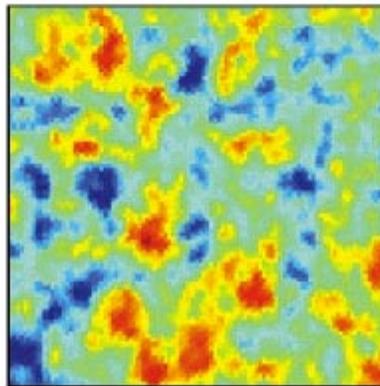
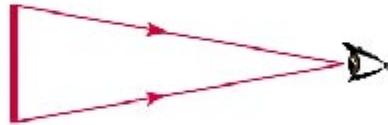
- **ZERO CURVATURE:** Two parallel beams of light never intersect – the universe is flat.
- **POSITIVE CURVATURE:** Two initially parallel beams of light gradually converge – the universe is spherical and is closed.
- **NEGATIVE CURVATURE:** Two initially parallel beams of light gradually diverge – the universe is hyperbolic and is open.



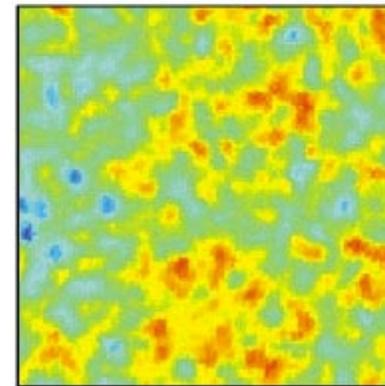
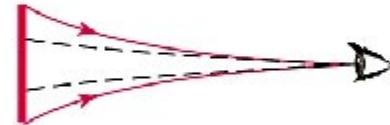
Tiny temperature variations in the Cosmic Microwave Background (CMB) are observed to be about 3×10^{-4} K.



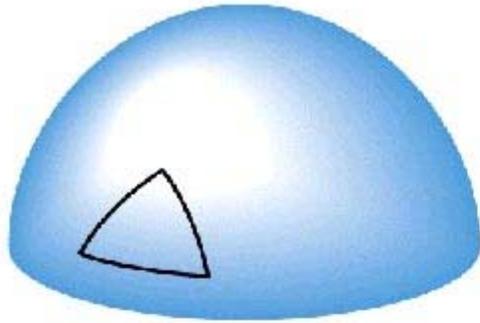
a If universe is closed, "hot spots" appear larger than actual size



b If universe is flat, "hot spots" appear actual size

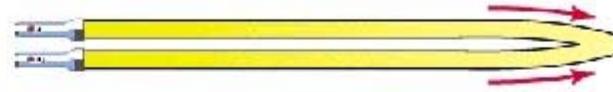


c If universe is open, "hot spots" appear smaller than actual size

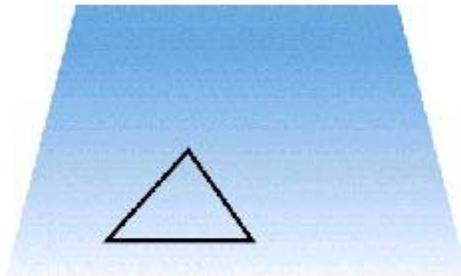


a Spherical space

$$\rho_0 > \rho_c, \Omega_0 > 1$$

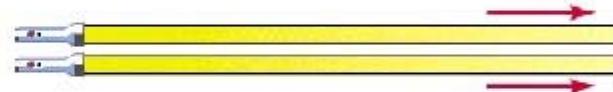


Parallel light beams converge

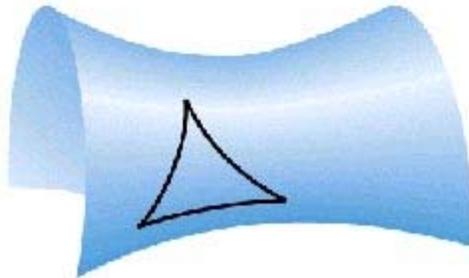


b Flat space

$$\rho_0 = \rho_c, \Omega_0 = 1$$

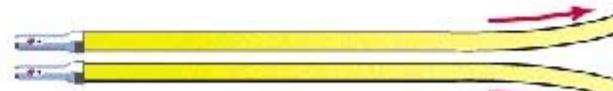


Parallel light beams remain parallel



c Hyperbolic space

$$\rho_0 < \rho_c, \Omega_0 < 1$$



Parallel light beams diverge

The shape of the universe indicates its matter and energy content.

Critical density of the universe

$$\rho_c = 3H_0^2 / 8\pi G$$

- H_0 is the Hubble constant and G is the universal constant of gravitation.

Density parameter Ω_0

$$\Omega_0 = \rho_0 / \rho_c$$

- ρ_0 is the combined average mass density.

*For $H_0 = 70 \text{ km/s/Mpc}$,
 $\rho_c = 9.2 \times 10^{-27} \text{ kg/m}^3$*

The shape of the universe indicates its matter and energy content.

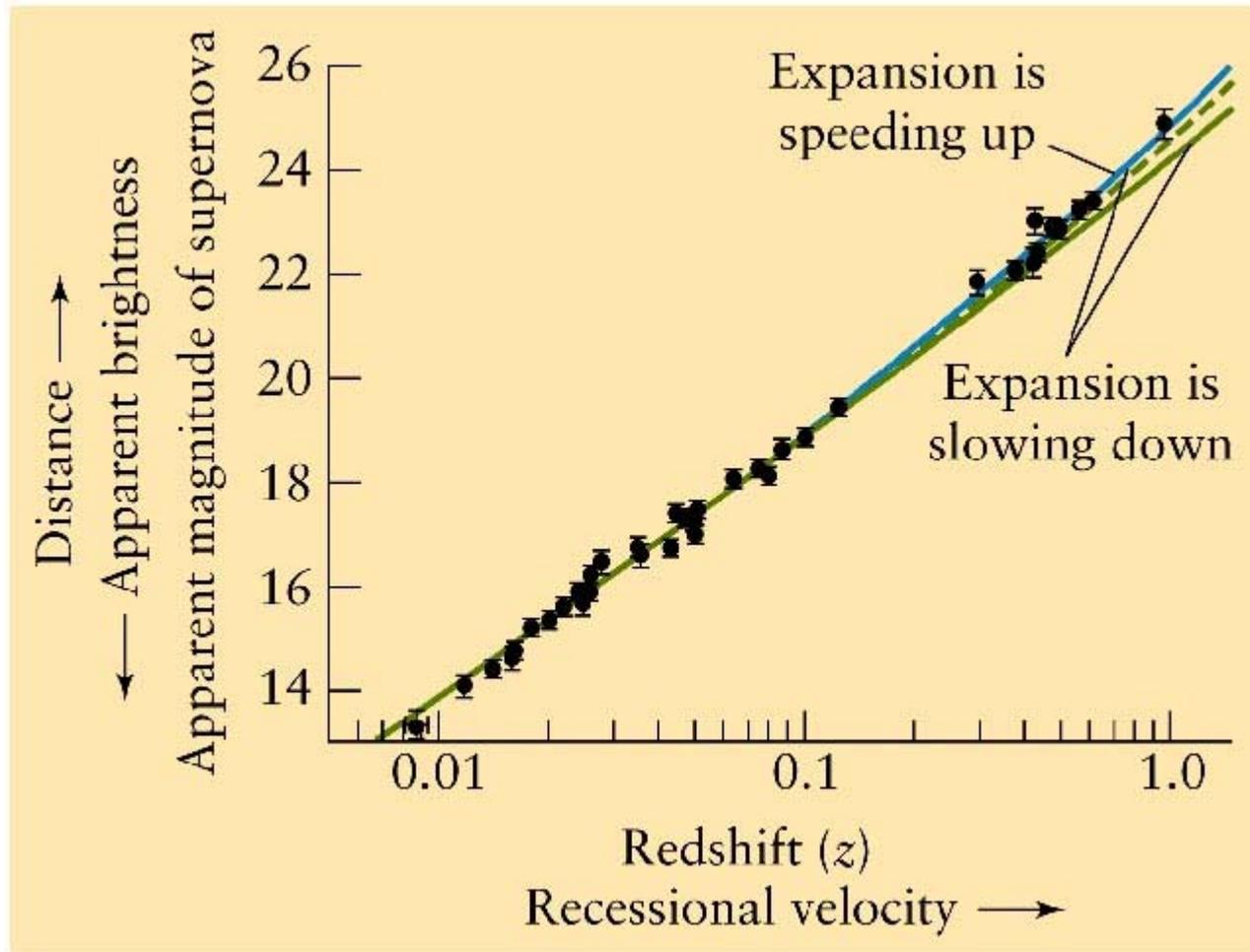
Table 28-1 The Geometry and Average Density of the Universe

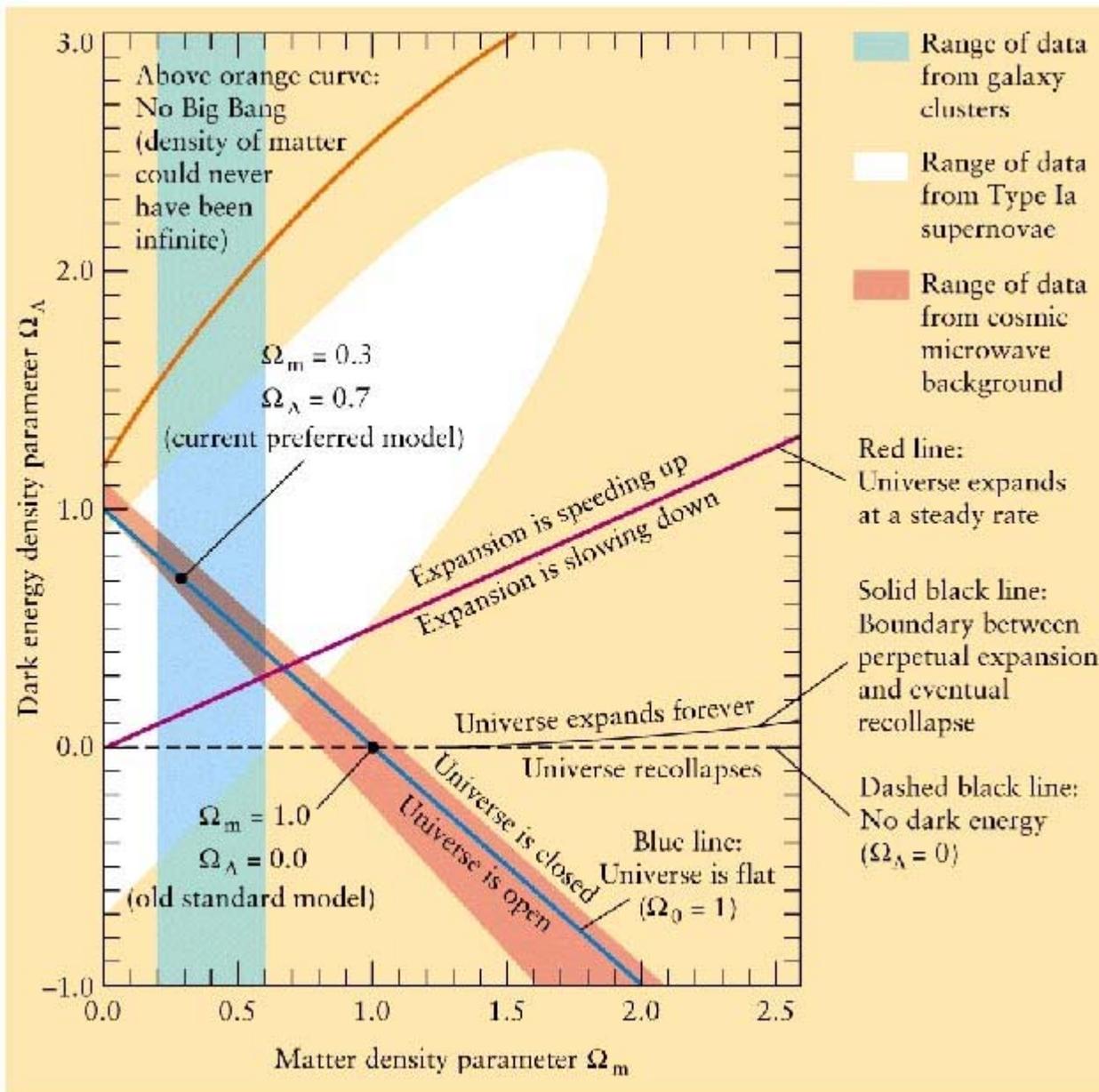
Geometry of space	Curvature of space	Type of universe	Combined average mass density (ρ_0)	Density parameter (Ω_0)
Spherical	positive	closed	$\rho_0 > \rho_c$	$\Omega_0 > 1$
Flat	zero	flat	$\rho_0 = \rho_c$	$\Omega_0 = 1$
Hyperbolic	negative	open	$\rho_0 < \rho_c$	$\Omega_0 < 1$

The universe appears to be filled with dark energy.

- Our observations suggest that the universe is flat.
- This conflicts somewhat with our observation that all known radiation, matter, and dark matter only account for 20% to 40% of the total density of the universe.
- There must be an additional source somewhere – dark energy.

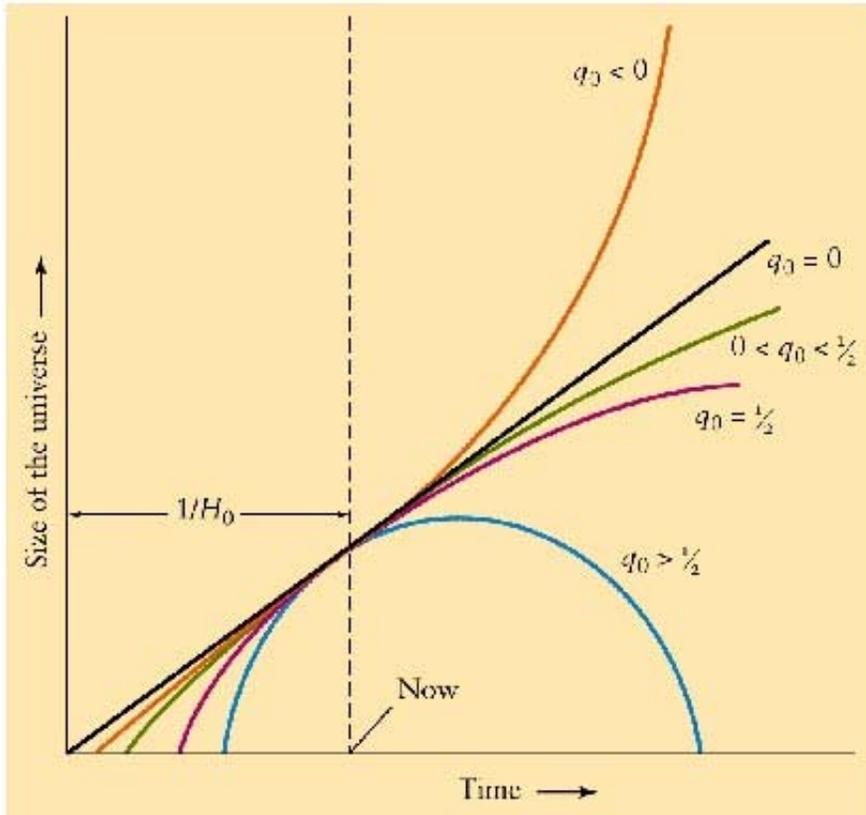
Observations of distant supernovae indicate that we live in an accelerating universe.





The matter and dark energy in the universe determine its future evolution.

Deceleration parameter (q_0)



- If $q_0 = 0$, then the universe expands forever at a constant rate.
- If $q_0 = 1/2$, then the universe is marginally bounded and just barely is able to continue expanding.
- If $q_0 < 1/2$, then the universe is unbounded
- If $q_0 > 1/2$, then the universe is bounded and will eventually collapse in on itself ending in a *big crunch*.

Four Models for the Future of the Universe

1. Recollapsing Universe: the expansion will someday halt and reverse
2. Critical Universe: will not collapse, but will expand more slowly with time
3. Coasting Universe: will expand forever with little slowdown
4. Accelerating Universe: the expansion will accelerate with time

