1 Anisotropies

1.1 Angular scale, ℓ-value

The multipole value can be approximated as

$$\ell \approx \frac{\pi}{\theta} \tag{1}$$

where θ is the angular size in radians. We also know that the relationship between the proper size, D of an object and its (small) angular size θ is given by

$$\theta = \frac{D}{d_A} \tag{2}$$

where d_A is the angular distance, which can be written as

$$d_A = \frac{d_L}{y^2} = \frac{1}{H_0 y}\int_0^y [1 - \Omega_M + \Omega_M y'^3]^{-1/2} dy' \tag{3}$$

where d_L is the luminosity distance and $y = z + 1$.

This gives the angle

$$\theta = \frac{D H_0 y}{\int_0^y [1 - \Omega_M + \Omega_M y'^3]^{-1/2} dy'} = \frac{D_0 H_0}{\int_0^y [1 - \Omega_M + \Omega_M y'^3]^{-1/2} dy'} \tag{4}$$

where D_0 is the co-moving size.

For $D_0 = 5h^{-1}$ Mpc we obtain $\theta \approx 3$ arcminutes and thus $\ell \approx 4000$.

1.2 Anisotropies, $\Delta T/T$

For a spectrum with $T \approx T_0 + \Delta T$:

$$B(T) \approx B(T_0) + \left.\frac{dB}{dT}\right|_{T_0} \Delta T \tag{5}$$

where T_0 is the CMB temperature today. From our draft we have

$$i_\nu = \left.\frac{dB_\nu}{dT}\right|_{T_0} T_0 \int_0^{\tau_\nu} \frac{T_d(z) - T_{CMB}(z)}{T_{CMB}(z)} d\tau_e. \tag{6}$$

Comparing these two and supposing a 10% variation between a dark matter filament and the surrounding void, we obtain

$$\frac{\Delta T}{T_0} = \frac{0.1 \cdot i_\nu}{T_0 \left.\frac{dB_\nu}{dT}\right|_{T_0}} = 0.1 \cdot \int_0^{\tau_\nu} \frac{T_d(z) - T_{CMB}(z)}{T_{CMB}(z)} d\tau_e. \tag{7}$$

which we evaluate to

$$\frac{\Delta T}{T} \approx 7 \times 10^{-4} \tag{8}$$

when integrating from $z = 0$ to $z = 20$ and supposing $f_d = 0.3$ and $\Delta t = 1$ Gyr. For $f_d = 0.1$ and $\Delta t = 0.1$ Gyr, we get $\frac{\Delta T}{T} = 4 \times 10^{-5}$.